Outline

- Timetable Development
- Fleet Size Calculation
- Vehicle Scheduling

Timetable Development

If we have $N$ departures in the peak period:

- **Equal headway solution**
  \[ H = \frac{\text{Peak Period Duration}}{N} \]

- **Balanced load solution**
  \[ \text{Load} = \frac{\text{Total Passenger Flow}}{N} \]

More vehicles might be required for a balanced load solution.

Fleet Size Required

**Salzborn's Fleet Size Theorem**

Given:

- $l(k, t, s) = \#$ of departures from terminal $k$ by time $t$ following schedule $s$
- $a(k, t, s) = \#$ of arrivals at terminal $k$ by time $t$ following schedule $s$

and:

- $d(k, t, s) = l(k, t, s) - a(k, t, s)$
  
  deficit function at terminal $k$ at time $t$ following schedule $s$

**Assumptions**

- no trip shifting
- no deadheading (to balance deficit among terminals)
Fleet Size Required

Salzborn’s Fleet Size Theorem

Then:

\[ N(s) = \sum_{k \in K} \max_t (d(k, t, s)) \]

Where \( K \) is the number of terminals.

Vehicle Scheduling Problem

Objective

- Define vehicle blocks (sequences of revenue and non-revenue activities for each vehicle) covering all trips so as to:
  - minimize fleet size
  - minimize non-revenue time and mileage

Observation

- These are proxies for cost, but a large portion of cost will depend on crew duties which are unknown at this stage of solution.

The deficit function, or minimum required fleet size, may be reduced by:

- shifting departure and/or arrival times
- adding deadhead trips between terminals

Input

- A set of vehicle revenue trips to be operated, each characterized by
  - starting point and time
  - ending point and time
- Possible layover arcs between the end of a trip and the start of a (later) trip at the same location
- Possible deadhead arcs connecting
  - depot(s) to trip starting points
  - trip ending points to depot(s)
  - trip ending points to trip starting at a different point
Vehicle Scheduling Problem

Observations

- there are many feasible but unattractive deadhead and layover arcs, so it is best to generate only plausible non-revenue arcs
- layover time affects service reliability, so set minimum layover (recovery) time

Variations

- each vehicle restricted to a single line vs. interlining permitted
- single depot vs. multi-depot
- vehicle fleet size constrained at depot level
- routes (trips) assigned to specific depot
- multiple vehicle types

Time-Space Network Representation

- revenue arc
- layover arc
- deadhead arc

Depot

Route 1

A₁ B₁

Route N

Aₙ Bₙ

(time of day)

Depot

(time of day)

Depot
The vehicle scheduling problem can be modelled as a *minimum cost network flow problem*, with arcs representing trips.

- Arcs have lower and upper bound constraints on flow
  - revenue arc $l = 1, u = 1$
  - layover arc $l = 0, u = 1$
  - deadhead arc $l = 0, u = 1$

- Arcs have cost
  - revenue arc cost irrelevant
  - layover arc driver cost of extra layover time
  - deadhead arc driver & vehicle cost of deadhead
Mathematical Modelling: Network Flows

- **Vehicle Scheduling Problem (VSP)**
  - find a set of feasible vehicle blocks such that
    - each trip in the timetable is covered exactly once
    - total cost is minimized
    - layover arcs
    - deadhead arcs
    - maximum block length
    - flow conservation
  - significant cost reductions with interlining and trip shifting heuristics
- **Single Depot Vehicle Scheduling Problem (SDVSP)**
  - for smaller agencies
  - solvable in polynomial time (minimum cost network flow)
- **Multidepot Vehicle Scheduling Problem (MDVSP)**
  - for large agencies
  - (integer multicommodity flow)
  - NP-hard (trip-depot compatibility constraints)
  - exact algorithms exist, but heuristics are used in practice
  - SDVSP used to find suboptimal solution, or as a sub-problem

Heuristic Approaches

1) Define compatible trips at same terminal $k$ such that trips $i$ and $j$ are compatible if and only if:
   - $M_k < t_{ij} - t_{ei} < 2D_k$
   
   where
   - $t_{ij}$ = starting time for trip $j$
   - $t_{ei}$ = ending time for trip $i$
   - $M_k$ = minimum recovery/layover time at terminal $k$
   - $D_k$ = deadhead time from terminal $k$ to depot

2) Apply restricted first-in-first-out rules at each terminal
   - **Step a)** Start with (next) earliest arrival at terminal; if none, go to step (d)
   - **Step b)** Link to earliest compatible trip departure; if none, return vehicle to depot and return to step (a)
   - **Step c)** Check vehicle block length against constraint:
     i) if constraining, return vehicle to depot and return to step (a)
     ii) otherwise, return to step (b) with new trip arrival time
   - **Step d)** Serve all remaining unlinked departures from depot

Network Representation (MDVPS)

- Conservation of flow at depots


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Single Route Scheduling Practice

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<th>AM Peak Period</th>
<th>Base Period</th>
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<tr>
<td>Headways</td>
<td>6:00 - 9:00</td>
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<td>Scheduled Trip Time</td>
<td>A→B or B→A</td>
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<td>Minimum Layover Time</td>
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Dominant direction of travel in AM Peak is A→B

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**Results of earlier planning and scheduling analysis:**
### Timetable and Vehicle Block Development

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x = pull out (from depot)  y = pull in (back to depot)
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### Timetable and Vehicle Block Development

**Block 1:** Depot - A (6:00) - B (6:50) - A (7:40) - B (8:30) - A (9:30) - B (10:15) - A (11:00) - B (11:45) - . . .

**Block 2:** Depot - A (6:20) - B (7:10) - A (8:00) - B (8:50) - Depot

**Block 3:** Depot - A (6:40) - B (7:30) - A (8:20) - B (9:15) - A (10:00) - B (10:45) - . . .

**Block 4:** Depot - A (7:00) - B (7:50) - A (8:40) - Depot

**Block 5:** Depot - A (7:20) - B (8:10) - A (9:00) - B (9:45) - A (10:30) - B (11:15) - . . .