III-2 STRESS-STRAIN-STRENGTH PROPERTIES

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Sheets
A: C10(1) data on loose dense sand on F0

B: Particle crushing
C1: State Parameter \( \gamma \)
D: Bottem (1986)
E1-E4: MIT-31 model / shear data

F: Effect of \( b \) / sample preparation
G: Anisotropy data
H: \( K_0 \) data
PART III - 2 STRESS-STRAIN-STRENGTH PROPERTIES (p2)

1. INTRODUCTION

1. If soil were linear-elastic-isotropic with infinite strength, then one simple test → 2 elastic constants to completely define σ-ε characteristics.

2. But soil is "particulate" system of finite strength wherein plastic strains result from:
   a) Deformation of particles - elastic & crushing (granular)
   b) Sliding & rolling amongst particles

3. Soil Mechanics, therefore, developed several types of tests that attempt to simulate typical conditions encountered in the field. (Long before existence of reliable soil models)

2. THREE MOST COMMON TESTS

2.1 1-D Consolidation (Compression) = Oedometer Test (For stress-strain)

1. Field situation
   ![Diagram of field situation]
   Loaded area is large with soil thickness; min. n → 5

2. Lab test
   ![Diagram of lab test]
   \[ P/A = q' \]
   \[ D/H = 3-4 \]
   \[ 60-75 \text{mm} \]

3. Typical σ-ε
   ![Graph of σ-ε relationship]
   - Strain hardening
   - Plastic deformations
   - Constrained modulus
   \[ D = \frac{\Delta q'_{v}}{\Delta e_{v}} = 1/m_{v} \]
   (Coef. of volume change)

4. "Elastic" relationships
   \[ D = \frac{E'(1-\mu')}{(1+\mu')(1-2\mu')} \]
   \[ K_{0} = \mu'/(1-\mu') = 0.50 \text{ for } \mu' = 1/3 \]
PART III-2. STRESS-STRAIN-STRENGTH PROPERTIES (p3)

2.2 DIRECT (Box) Shear Test (one of 1st strength tests)

(i) Field situation

(ii) Lab test

(iii) Typical $\tau_h$ - "strain"

Medium-dense sand

2.3 TriaXial Test (1st used soils 1930s; both $\sigma$-$e$ & strength)

(i) Field situation

(ii) Lab test (cylindrical sample)

(iii) Typical $\sigma$-$e$
2.3 Triaxial Test (Continued)

(a) Four basic types of tests (drained starting from isotropic \(q'_c\))

\[
q = \frac{(\sigma_v - \sigma_h)}{2}
\]

\[
K^r = \frac{\sigma_h}{\sigma_v'}
\]

\[
K^l = \frac{\sigma_h}{\sigma_v'}
\]

\[
\frac{1}{K^l} = \frac{\sigma_h}{\sigma_v'}
\]

- Triaxial Compression (\(\sigma_2 = \sigma_3 = \sigma_h\))
  - \(\sigma_v > \sigma_h\) (\(K < 1\))
  - Axial (vertical) compression of sample
  - \(b = \_\)

- Triaxial Extension (\(\sigma_2 = \sigma_3 = \sigma_h\))
  - \(\sigma_h > \sigma_v\) (\(K > 1\))
  - Axial (vertical) extension of sample
  - \(b = \_\)

<table>
<thead>
<tr>
<th>Letter</th>
<th>(\Delta \sigma)</th>
<th>Description</th>
<th>Field Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(+ \Delta \sigma_v)</td>
<td>(\Delta \sigma_h = 0)</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>(\Delta \sigma_v = 0)</td>
<td>(- \Delta \sigma_h)</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>(\Delta \sigma_v = 0)</td>
<td>(+ \Delta \sigma_h)</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>(- \Delta \sigma_v)</td>
<td>(\Delta \sigma_h = 0)</td>
<td></td>
</tr>
</tbody>
</table>

- Can, of course, vary both \(\sigma_v\) \& \(\sigma_h\) during testing
- \(\sigma_2\) condition defined by \(b = \frac{(\sigma_2 - \sigma_3)}{(\sigma_1 - \sigma_2)} = \_\) TC
  \(\frac{(\sigma_1 - \sigma_3)}{(\sigma_1 - \sigma_2)} = \_\) TE
3. STRENGTH OF COHESIONLESS SOILS (At "Low" Confinement)

3.1 Mohr-Coulomb Failure Criteria

1) Std. TC tests at varying $\sigma'_c$ on med-dense sand (Drained, $\sigma' = \sigma$)

\[
\sigma_c = \sigma_{3f}
\]

\[
\phi = \phi'
\]

\[
\theta_{cr} = \gamma
\]

\[
\tau_{ff} = \sigma_{ff} \tan \phi'
\]

\[
\theta_{cr} = \theta
\]

2) Mohr-Coulomb failure criteria states:

No.1 Envelope represents limiting condition of state of stress, cannot have SOS for which Mohr circle lies above envelope.

No.2 When Mohr circle tangent to envelope, then point of tangency represents conditions on failure plane = rupture surface, where shear stress = shear strength, leading to large deformations.

Note: See Table III.2-1 for some useful equations (p.5a)
Table III 2-1 Equations for Computing Stresses with Mohr Circle

Note: θ is angle between plane and σi plane (RF1)

Definitions & Identities

1. \[ N_\phi = \frac{1 + \sin \phi}{1 - \sin \phi} = \tan^2(45 + \frac{\phi}{2}) \] \[ \text{For } \phi = 0, \text{ } R_F = \frac{(\sigma_1 + \sigma_2)}{2} = N_\phi ; \sin \phi = \frac{R_F - 1}{R_F + 1} \] \[ \text{Eq. No. 2} \]

2. \[ \sqrt{N_\phi} \cdot \tan(45 + \frac{\phi}{2}) = \frac{\cos \phi}{1 - \sin \phi} \] \[ \frac{1}{\sqrt{N_\phi}} = \tan(45 - \frac{\phi}{2}) = \frac{\cos \phi}{1 + \sin \phi} \] \[ \text{Eq. No. 4} \]

For Any State of Stress

3. \[ \tau_{max} = \tau_m = 0.5(\sigma_1 - \sigma_3) \] \[ \tau_o = \tau_m \sin 2\theta = 2\tau_m \sin \theta \cos \theta \] \[ \text{Eq. No. 6} \]

4. \[ \sigma_{mean} = \sigma_m = 0.5(\sigma_1 + \sigma_3) \] \[ \sigma_o = \sigma_m + \tau_m \cos 2\theta = \sigma_1 \cos^2 \theta + \sigma_3 \sin^2 \theta \] \[ \text{Eq. No. 8} \]

For States of Stress at Failure

5. \[ \tau_{ff} = \tau_m \cos \phi = C + \sigma_{ff} \tan \phi \] \[ \text{Eq. No. 10} \]

6. \[ \sigma_{ff} = \tau_{ff} \tan \phi = \sigma_m - \tau_m \sin \phi \] \[ \text{Eq. No. 12} \]

7. \[ \sigma_{ff} = \sigma_{ff} + \sqrt{N_\phi} \tau_{ff} = \sigma_3 \cdot N_\phi + 2C \sqrt{N_\phi} \] \[ \text{Eq. No. 14} \]

8. \[ \sigma_3 = \sigma_{ff} - \frac{\tau_{ff}}{\sqrt{N_\phi}} = \frac{\sigma_{ff}}{N_\phi} - \frac{2C}{\sqrt{N_\phi}} \] \[ \text{Eq. No. 16} \]
3.2 Presentation of Triaxial Test Data \((\sigma = \sigma')\)

\[ \frac{\sigma_f}{P_f} = \tan \phi' = \sin \phi' \]

For ease of presentation, NOT different failure criteria.

3.3 Interpretation of Direct Shear Test (Really indeterminate)

(i) Usual assumption of horizontal failure plane, i.e., \(\tau_h = \tau_f\) \((\sigma = \sigma')\)

- For NC sand starting from \(K_0\) condition

\[ \begin{align*}
\sigma_v &= \sigma_{ic} \\
\tau_h &= \sigma_v (= \sigma_{ic}) \\
\tau_f &= \sigma_{ff}
\end{align*} \]

\[ \Delta L/L_0 \]

Rotation \(\sigma_1\)
direction =

(ii) Alternative assumption that \(\tau_h (\text{max}) = \tau_{\text{max}} (\sigma = \sigma')\)

\[ \phi_i = \arctan \frac{\tau_h}{\sigma_v} \{ \text{Common} \}
\]

\[ \phi_2 = \arcsin \frac{\tau_h}{\sigma_v} \{ \text{Conservative} \}
\]

\((\tau_h/\sigma_v = 0.6 \rightarrow \phi' = 31^\circ \text{vs. } 37^\circ)\)
3.4 Effect of Relative Density (Illustrated via Std. TC tests)

1. Stress-strain data \( (\sigma = \varepsilon^t) \)
   - Danse
   - Loose \( \{ \sigma'_c = \sigma'_f = 1 \text{ atm} \}

\[
\frac{\sigma'_c - \sigma'_3}{\sigma'_c} = R - 1
\]

\[V = \frac{\Delta V}{V_0}\]

- Small \( \varepsilon^t \)
- Significant strain softening
- Initial small contraction, then
  - Large expansion (dilation)

**Loose**
- Large \( \varepsilon^t \)
- Little strain softening

For both Critical = Steady State

\[\times \text{ Unique } \sigma - q - p \text{ condition} \]
\[\\times \text{ with continued shearing} \]

[Called Critical State Line = Steady State Line]

\[
R = \left( \frac{\sigma'_c}{\sigma'_3} \right) = \tan^2 \left( 45 + \phi'_t/2 \right)
\]

\[
= \left( \frac{1 + \sin \phi'}{1 - \sin \phi'} \right)
\]

\[
\phi'_c = \phi'_s = \phi'_t \]

Also \( \sin \phi' = (R - 1)/(R + 1) \)

3.5 Three Components of Strength (Rowe, 1962; differs from Li'W)

1. Frictional resistance

\[
N
\]

\[
T \left\{ \begin{array}{c}
\phi'_t
\end{array} \right\}
\]

- Coef. of friction \( \mu = T/N = \tan \phi'_t \)
- Rowe (1962) states that \( \phi'_t \)
  - due to sliding only
- But more recent research
  - indicates that also rolling at high \( \phi'_t \)
  - (Skinner 1969, Geot.)
(2) **Resistance due to Dilation**

- Component due to expansion of soil during shear against the confining stresses (expansion from "interlocking")

  - Magnitude is proportional to rate of volume change

    \[ R_p = \left( \frac{C'_p}{C'_3} \right)_{\text{max}} (1 + RD) \tan^2 (45 + \frac{\phi'_I}{2}) \]

  \[ R_f = \text{MEASURED BACKCALCULATED} \]

  \[ \text{Expansion} \rightarrow \frac{\Delta V}{V_0} = \nu \]

  \[ \text{Compression} \rightarrow \frac{\Delta V}{V_0} = \frac{\Delta \varepsilon}{\varepsilon_0} \]

  \[ \text{Slope} = -\frac{\Delta V}{\Delta \varepsilon} = RD \]

  \[ \phi'_p \text{ occurs at max, slope} \]

(3) **Resistance due to Interference**

- "Interlocking" also results in fact that sand particles cannot move in a straight line, but must go around each other

- At constant vol. \( (dv/d\varepsilon = 0) \)

  \[ \tan \phi'_c = \frac{\mu}{2} \tan \phi'_I \]

  \[ (\frac{\mu}{2} \text{ circumference/diameter}) \]

  (really not that simple)

(4) **Summary**

- Very dense: \( \phi'_p = \phi'_c + \phi'_d \)

- At critical state and very loose: \( \phi'_p = \phi'_c = \phi'_I + \phi'_d \)

- Intermediate: \( \phi'_p = \phi'_c \times \phi'_I \times \phi'_d \)

  IHW

  "Interlocking"

\[ \phi' = 28 \pm 20 \text{° Quartz} \]

\[ \phi'_p \text{ calculated from measured } \phi'_p (\text{ie. } R_{\text{max}} + \max [-dv/d\varepsilon]) \]
4. COMBINED EFFECTS OF DENSITY AND CONFINING PRESSURE ON STRENGTH OF GRANULAR SOILS

4.1 Overview of Data From Standard Triaxial Compression Tests

1) Effect of confining stress level on stress-strain behavior of dense sand (idealization of data shown in Fig. 3 of Sheet A)

\[ \frac{(\sigma'_1 - \sigma'_3)}{\sigma'_c} = R - 1 \]

\[ (E \propto \sqrt{\sigma'_c}) \]

\[ \nu = \frac{\Delta v}{v_0} \ (%), \ \sigma'_c \]

Trends for Increasing \( \sigma'_c \):
- Increasing \( E_f \) (at peak)
- Less expansion (dilation) to only compression (contraction)
- Lower mean rate of dilation (\( R^0 = -d\nu/d\sigma'_c \))
- Decreasing \( \phi'_p \)

...Same basic trends as effect of decreasing \( D_r \) at low \( \sigma'_c \)

2) Peak friction angle (\( \phi'_p \)) vs. Confining stress at failure (see Fig 3.2-1, p 9a)

- See large variation in "pressure sensitivity" (\( d\phi'_p/d\log \sigma'_c \))
  - for different densities and types of granular soil
- In general, larger pressure sensitivity
  - With increasing \( D_r \), e.g. data of Lee and Fissed (1967) and van Cogh (1968)
  - With weak sand grains, e.g. Ottawa sand > calcareous sand (quartz)
- Therefore related to compressibility of test material

Note: \( d\phi'_p/d\log \sigma'_c \) \( \approx 5-10^\circ \) for rockfill, very dense sand, gravel and for calcareous soils (even when loose, see line 0 in Fig. 3.2-1)
Fig. III.2-1 Effect of Stress Level on Peak Friction Angle of Loose and Dense Granular Materials
3) Particle crushing

![Graph showing particle crushing with different stages: initial, after shearing, and after consolidation.]

(See Sheet B for actual data)

Dense Ottawa: little crushing at $Q_e = 40$ ksc and low pressure sensitivity

Dense Chattahoochee:
Sacramento River Sands: a lot of crushing and high pressure sensitivity

4) Strength component of dense sand with increasing confinement

[Modified version of Lee & Seed (1967)]

\[ R_p = \tan^2 (45 + \phi'_p/2) \]

\[ R_f = \tan^2 (45 + \phi'_f/2) \]

\[ R_p = (1 + RD) R_f \] (Rowe, 1962)

**Notes:**
1. $\phi'_p = \text{interference + particle crushing}$
2. At very high $Q_s$, may require very large shear to reach CSL = SSL (e.g., see Fig. 5.27, Sheet E3)

Will next look at three different approaches for evaluating the combined effects of density and confinement on shear-strength of sands.
4.2 State Parameter, $y$ (Been & Jefferies, 1985; Been et al., 1990)

1) Definition

$$y = \psi = C - C_{ss}$$. At same $\sigma_{out}$, i.e., SSL = reference state

Blue $\psi_A$ : $\psi_A \to$ contractile behavior
Negative $\psi_B \to$ dilative behavior

Pt A + B : effect of increasing $C$:
Pt B + C : $\ldots$ $\ldots$ $\sigma'$

Shear at constant volume:
steady state line (SSL) =
critical state line (CSL)

$$\log \sigma'_{out} \left[ \frac{1}{3} (\sigma'_1 + \sigma'_2 + \sigma'_3) \right]$$

2) Examples of "unique" correlation between $\psi$ and shear behavior

See Sheet C1 for actual correlation:

- Fig 14: Decreasing $\psi \to$ increasing rate of dilatation at peak strength
- Fig 15: $\ldots$ $\ldots$ $\phi'_p = \phi'_C$ ($\phi'_C = \phi'_C$)
- Fig 16: $\ldots$ $\ldots$ $\phi'_p$

3) Problems with application in practice

a) Slight variations in particle size distribution can have a large effect on location of the SSL, e.g., Sheet C1, Fig. 7 for uniform medium sand with 0.2, 5%, 10% fine (#100)

b) Whether or not drained or undrained shear from $\pm \psi$ will produce the same SSL = CSL is still controversial. Sheet C2, Fig. 12 indicates that drained shear with $-\psi$ does not reach SSL (maybe due to "non-uniform conditions (shear planes) in hot specimen"

c) Get marked curvature in CSL at shear $\to$ significant crushing, e.g., Sheet C2, Fig. B. 11 $\frac{7}{12}$

CCL Conclusion: Excellent concept (especially for teaching), but difficult to use quantitatively in practice.
4.3 Semi-Empirical Correlations (Bolton 1986)

1) Approach: Evaluated drained $TC_a$ shear data from 17 field programs (Sheet D, Table) to determine effects of $D_r$ and $\phi'$ on max rate of dilation and especially $\Delta \phi' = \phi_p' - \phi_s'$ ($\phi_s' = \phi_s^0$).

2) Results of study (for tests that dilate during shear, i.e., start with $\gamma < 0$) led to $I_R = \text{relative dilatancy index}$ (Note: at $\sigma' = \sigma_{cat}$, $\phi'$ is not the same as $\phi$)

$$I_R = D_r \left( 10 - \ln \sigma_r' \right) - 1$$

where $D_r = \text{relative density (decimal)}$

$\sigma_r' = \sigma_{cat}$ at failure in kPa

3) Resulting correlations for Std. $TC$ (C10C12)

$$\Delta \phi' = \phi_p' - \phi_s' = 3 \cdot I_R^0 \text{ and } \text{Max RD} = 0.3 I_R^0$$

(Note: For plane strain, $\Delta \phi' = 5 \cdot I_R^0$)

4) Examples of predictions vs. measured data (See Sheet D)

- Fig. 7 Effect of increasing $D_r$ on $\Delta \phi'$ and max $\text{RD}$ at $\sigma_{cat} = 300$ kPa (both $TC'/PS$)

- Fig. 9 Effect of increasing $D_r$ on $\Delta \phi'(TC)$ at $\sigma_{cat} = 20, 50, 100, 600$ kPa (both $TC'$)

- Fig. 10 Effect of increasing $\sigma_{cat}$ at varying $D_r$ on $\Delta \phi'(TC)$

5) Bolton (1986) also suggests that: Typical errors in $D_r = \pm 5\%$; $\phi_s' = 33^\circ \pm 1^\circ$ for mainly quartz sand; $\phi_s' = 40^\circ$ for feldspar grains

6) Pedestis (1996) concludes from results in Section 4.4 that $\Delta \phi'(TC)$ is measurable at $\sigma_{cat} > 100$ kPa, except when $D_r = 100\%$. But $\Delta \phi'(PS)$ is too high.

1) Background on formulation for granular soils

- Use Limiting Compression Curve (LCC) = linear portion of \( \frac{\log a_0}{\log \sigma_{at}} \) compression curve at high stresses where particle crushing predominates as the reference state (also VCL for clays).

  See Sheet E1, Fig. 2.2 125

- For shear behavior, start with "basic" elasto-plasticity theory (à la MCE), but adds a lot of new features to incorporate hysteresis, anisotropy, strain softening, etc. See Sheet E3, Fig. 4.8

2) Input parameters for Toyoura Sand and some remarks

a) See Sheet E2, Table 5.2 for testing to obtain 14 parameters. Key tests are:

- Compression test to high \( \sigma_c' \) to define location of LCC, with UH cycle (hysteresis) and values of \( K_0 \)
- Undrained TC test from \( \sigma_c' \) on LCC \( \rightarrow \) shape of drainage surface, \( D_p \)
- Drained TC test at low \( \sigma_c' \) \( \rightarrow \) \( P_k \), etc.
- Resonant column test to get small strain stiffness

b) Sketch of Sheet E2, Fig. 5.4 5.24

Using input data from a test at one \( \sigma_0 \), predicts compression and undraining at all values of \( \sigma_0 \)

Also predicts location of the critical state line (CSL)!
3) Prediction of drained TC shear behaviour of Toyama Sand

a) Effect of varying $D_r$ at $\sigma'_c = 100$ kPa ($\approx 1$ atm)
   Sheet E3, Fig. 5.26

b) Effect of varying $\sigma'_c$ at $\sigma_0 = 0.8$ ($D_r = 452$)
   Sheet E3, Fig. 5.27

c) Combined effect of varying $D_r$ and $\sigma'_c$ on $\phi'$
   Sheet E4, Fig. 5.28  Note $\phi'$ linear vs $\sigma_0$ at constant $\sigma'_c$

d) Comparison with Bolton's (1986) empirical eqn for $\phi'$ and RD
   Sheet E4, Fig. 5.29  → rather remarkable agreement

NOTE: Most of the measured drained compression shear data on Toyama Sand
are from undrained shear tests. Comparison of predicted vs. measured behaviour is covered in 1.322 (Soil Behavior)
5. OTHER FACTORS AFFECTING THE STRENGTH OF GRANULAR SOILS

5.1 Intermediate Principal Stress ($\sigma_2$)

1) Field conditions leading to different values of $b = (\sigma_2 - \sigma_3) / (\sigma_1 - \sigma_3)$

- Circular Footing
  $\sigma_2 = \sigma_3 = \sigma_h$
  TC, $b =$

- Circular Excavation
  $\sigma_2 = \sigma_1 = \sigma_h$
  TE, $b =$

- Strip Footing
  Plane Strain since $\varepsilon_2 = 0$
  $\sigma_1 > \sigma_2 > \sigma_3$
  PS, $b = 0.25 - 0.4$

2) Device to measure the effect of varying $b = $ True Triaxial (very complex apparatus due to "come" conditions)

3) Some experimental results (Sheet F, Fig. 13)

- Conflicting data as $b \rightarrow 1$, probably due to experimental problem.
  Note: MIT-S1 used $\phi'_{TE} = \phi'_{TC}$
- All data show increase in $\phi_p$ as $b$ increases from zero (TC) to $b = \varepsilon_2 = 0$ (PS = plane strain)
- CCL recommends Bolton (1986). Therefore
  $\phi_p^{PS} - \phi_p^{TE} = 0$ for low Dr - high $\phi_{int}$ ( $\psi > 0$)
  = $5^o$ for high Dr - low $\phi_{int}$ ( $\psi < 0$)

\[ \sigma_1 \]
\[ \sigma_2 \]
\[ \sigma_3 \]
4) Comments: Bolton assumes same $\phi'_3$ for TC, RS, whereas Pastora (1990) concludes (caution) that PS $\rightarrow$ higher $\phi'_3$
   - Part III-4 will show that $\Delta \phi' = +5^\circ \rightarrow$ doubling of $q_{ult}$ (bearing capacity)

5.2 Method of Sample Preparation

1) Most lab shear tests on sand are run on reconstituted samples, the two most common methods are:
   - Pluviation, with or without vibration $\rightarrow$ more like natural deposits
   - Tamping (compaction) of moist sand $\rightarrow$ non-uniform density

2) Sheet F, Fig. 4 show example of very different shear-shear behavior (even though $\phi'_0$ = constant)

5.3 Anisotropy

1) 1-D deformation leads to a sand structure with
   - Preferred orientation of elongated grains 1 to $\sigma'_1$ (fabric)
   - ” ” ” particle contact (even with perfect spheres). See Sheet G, Fig. 8.15

2) Hence natural sand (deposit) has an inherent anisotropy wherein shearing at different $\phi$ angle leads to different stress-shear-strength properties
   - Shearing at $\phi = 0^\circ \rightarrow$ highest modulus $\phi'_p$
   - Increasing $\phi$ $\rightarrow$ lower modulus $\phi'_p$

3) Examples of trends (Sheet G)
   - Fig. 2 $\phi'_p$ vs. $\phi$ for several sands ($\Delta \phi' = 3 \pm 1^\circ$)
   - Fig. 8 Effect of $\phi$ on $E_v, E_v(L_1, L_2)$ $\forall$ $E_v$, for dense sand
6. 1-D BEHAVIOR OF GRANULAR SOILS

6.1 Data Presentation and Definition of Parameters

1) Introduction: Estimate settlement for 1-D loading

![Diagram](attachment:diagram.png)

Layer Thickness
\[ H_c \]
\[ \Delta \sigma' \]
\[ \sigma' \]
\[ \sigma_{vo} \]
\[ \sigma_{vf} \]

Settlement
\[ p = \Sigma (H_c \cdot \varepsilon_{ei}) \]

2) Conventional Methods of Plotting Oedometer Test Data

a) Linear plot

![Diagram](attachment:diagram.png)

- Strain hardening → decreasing slope with incr. \( \sigma' \)
- Coef. of compressibility, \( a_v = -d \varepsilon / d \sigma' \)
- Coef. of volume change, \( m_v = d \varepsilon / d \sigma' \) (more common)
- \( \varepsilon_v = \frac{\Delta \varepsilon}{1 + e_o} = \frac{a_v \Delta \sigma'}{(1 + e_o)} = m_v \sigma' \)

b) Semi-log plot (Note: \( d \log \sigma = \log_e d \sigma = 0.434 \frac{d \sigma}{\sigma} \))

![Diagram](attachment:diagram.png)

- Virgin compression index, \( C_v = -d \varepsilon / d \log \sigma' \)
- Virgin compression ratio, \( CR = \frac{C_v}{(1 + e_o)} = \frac{d \varepsilon_v}{d \log \sigma'} \)
- \( \varepsilon_v = \frac{\Delta \varepsilon}{1 + e_o} = \frac{C_v \log(\sigma'/\sigma_{vo})}{(1 + e_o)} = CR \log(\sigma'/\sigma_{vo}) \approx 0.434 CR \frac{\Delta \sigma'}{\Delta \sigma_{vo}} \)

NOTE: Because of difficulty of obtaining sand samples for lab testing, usually predict \( p \) from in-situ penetration tests (Part III-4.5)
6.2 Factors Affecting 1-D Compressibility

1) At stress significantly < $\sigma'_{VR}$: virgin loading; mostly plastic strain due to particle sliding & rolling

- Initial density, $D_r$
- Stress history, OCR
  - $RR < CR$ especially lower $D_r$
- Unload-reload cycles
  - Storage tanks, offshore platforms

Note: Pastana & Whittle (1995) conclude approximation $\log \sigma'_{VR} \approx \log \sigma'_{V}$ for virgin loading

2) At high stress → significant grain crushing (Pastana & Whittle 1995)
- Increasing $D_{50}$ → higher contact forces → lower $\sigma'_{VR}$
- Increasing $C_U$ (better gradation) → more rounded curve → higher $\Theta$
- Increasing angularity → lower $\sigma'_{VR}$ & higher $\Theta$
6.3 Coeff. of Earth Pressure at Rest \( K_0 \) BOTH Granular & Cohesive Soils

1) Lab Measurement Techniques
   a) Lateral stress oedometer
      - Closed system → problems with leakage & need \( DT \approx 0 \text{°C} \)
      - Results affected by side friction
   b) Automated stress path triaxial (MIT lab)
      - Load at constant \( E_a \) (inc. \( \sigma' \)) and vary \( \sigma'_h \) to maintain \( E_v = E_a \)

2) \( K_0 \) of NC (OCR = 1) Soils
   Jaky (1944) semi-emp. eqn. \( K_0 = 1 - \sin \phi' \)
   - For clays, \( K_0 = 0.45 - 0.7 \)
   - Eqn. here: \( SD = 0.05 \), quite good
   (See Sheet H, Fig. 3b)
   - For sands, \( K_0 = 0.4 \pm 0.1 \)
   - Eqn. doesn't fit data as well
   (See Sheet H, Fig. 14)

3) \( K_0 \) of OC Soils
   a) Unloading → higher \( K_0 \) (locked in \( \sigma'_h \))
      \( K_0 = K_{NC} (OCR)^n \), where \( n = 1 - K_{NC} \approx \sin \phi' \)
      - For clays, works quite well; for sands, less well
      (Sheet H, Figs. 15, 31, 32)
      - Max \( K_0 \) = failure in principal stress (shallow, clays)
   b) Reloading → significant hysteresis
      (Sheet H, Fig. 15f, 31)

4) References
   - Mauviel, Kulhawy (1982), JGED; 108(6): summary of "all" data in literature
PART III-2: STRESS-STRAIN-STR. PROP. (p. 20)

1. INFLUENCE OF STRESS PATH ON: STRESS VS. STRAIN

(1) Stress paths for O.C. dense sand (initial $k_0 = 1$)

- Isotropic compression
  $\Delta K = 1.0$

- I-D compression

- Std. triaxial comp.
  $\Delta K = 0$

(2) Resulting stress-strain curves

**Comments**

- Concave upward
  - $E_0 > E_{ot}$, since higher $p$
  - $h_0 > h_0$, due to anisotropy & higher $p$

- Upper hypoplastic loop to peak
  - $\frac{\Delta \sigma_v}{\sigma_{vo}} = \frac{E_a}{\sigma_{vo} (1 + b \cdot E_a)}$
  - Hence $E$ decreases with $\Delta \sigma$

- Man. (- ch/DE) at peak

- UR: Much stiffer response
  - (called evolving anisotropy)
  - (or kinematic hardening)