III-4 SHALLOW FOUNDATIONS ON SAND: BEARING CAPACITY

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Sheets A-D: Information on SPT procedures, \( N_0, N_1, f(D_r) \)
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Sheets G-I: Information on CPT correlations for \( D_r, \phi' \)
1. INTRODUCTION

1.1 Definitions

"Shallow Fdn = footing or mat with d/β ≤ 1"

1.2 Design Criteria

(a) Adequate Safety

\[ q_{allowable} = \frac{q_{ult}}{F} \]

Factors of Safety:

- \( F \approx 3 \) for Building
- \( F \approx 1.5 \) for Highway

(b) Allowable Settlement

Criteria (Part III-5) usually governs foundations on sand

Emphasis on prediction methods

2. ULTIMATE BEARING CAPACITY OF SOIL: THEORY

(Also see Vasic, 1973: JSMFD, ASCE Vol.99, 591)

2.1 Physical Model (Strip Footing)

\[ q_{ult} = \frac{q_{ult}}{\beta} \]

Neglect strength of soil above base

2.2 Shear Zones (Frictionless Base: approximate)

[Diagrams showing shear zones and slip lines]
Three Plastic Zones (States of failure drawn for c = 0)

I. Rankine Active
\[ q_{ult} = \sigma_1 f \]
\[ \sigma_3 f = \sigma_{ult} K_a \]

II. Rankine Passive
\[ q_s = \sigma_3 f \]
\[ \sigma_1 f = q_s K_p \]

II. Prandtl Radial

2.3 Resultant Solution: Strip Footing (Incompressible Soil)
\[ q_{ult} = C N_c + \frac{1}{2} Y B N_y + q_s N_b \]
with \[ q_s = \chi' d \]

Cohesion

Soil weight

Surcharge

where \[ N_c, N_y, N_b \] = bearing capacity factors
\[ = f(\phi) \]

For later in term:
\[ \text{Drained Shear} = f'(\sigma') \rightarrow c', \phi', \gamma'; \text{Undrained Shear} = f(\sigma) \rightarrow c, \phi, \gamma \]
2.4 Values of B.C. Factors (Vasic (1973) for details)

1. Theory of plasticity for rigid perfectly plastic soil

\[ N_c \neq N_g \]

For smooth base, where:

\[ N_g = \frac{1 + \sin \phi}{1 - \sin \phi} = \tan^2 \left( 45 + \frac{\phi}{2} \right) \]

\[ N_g = N_\phi \tan \phi \]

\[ N_c = \cot \phi (N_g - 1) \]

(Note: \( N_q = (Q_i/Q) \) for \( c = 0 \))

2. Value of \( N_q \) controversial since rigorous theoretical solution not available, and comparison of predicted vs. model footing test results inconclusive due to effects of:
   a) \( \sigma' \) level & \( \sigma_2 \) on value of \( \phi' \) of sands
   b) soil compressibility (\( \Delta v \neq 0 \))

Vasic (1973) recommends Casquet & Karlisel (1953):

\[ N_y = 2 \tan \phi (N_g + 1) \]

3. See Table III-4-1 (p5) for tabulated results (these differ from Fig 14.13 of LW).  

4. Some typical values

\[ \phi^0 = 0 \]

\[ N_c = 5.14 \]

\[ N_g = 1.00 \]

\[ N_y = 0 \]

\[ N_q/N_y = 0.8 \quad 0.7 \quad 0.6 \]

* For undrained shear of saturated soil, \( \phi = 0 \) if \( c = 0 \); \( N_c = \pi r^2 \)
### Table 4. Bearing Capacity Factors

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<th>φ (°)</th>
<th>$N_φ$ (kN/m²)</th>
<th>$N_q$ (kN/m²)</th>
<th>$N_y$ (kN/m²)</th>
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\[
N_φ = \frac{1 + \sin φ}{1 - \sin φ} = \tan^2 \left( \frac{45 + φ}{2} \right)
\]

\[
N_q = N_φ \cdot \pi \cdot \tan φ
\]

\[
N_c = \cot φ \cdot (N_q - 1)
\]

\[
N_x = 2 \tan φ \cdot (N_q + 1)
\]

\[
S_q = 1 + \tan φ \left( \frac{B}{L} \right)
\]

\[
S_c = 1 + \frac{N_q}{N_c} \left( \frac{B}{L} \right)
\]

\[
S_y = 1 - 0.4 \left( \frac{B}{L} \right)
\]
2.5 Illustration of Results: Strip Footing on Dry Sand

\[ Q_{\text{net}} = q_{\text{ult}} \cdot B - (W\text{gt. footing} + \text{soil}) \]

\[ q_{\text{ult}} = \frac{1}{2} \gamma B N_{q} + \gamma c d N_{q} \]

\[ = \frac{1}{2} \gamma B N_{q} \left(1 + \frac{2d}{B} \frac{N_{q}}{N_{q}}\right) \quad \text{for constant } \gamma \]

\[ \phi' \]

\[ d = 0 \quad d = B \]

30 \[ 11 \gamma B \quad +18 \gamma B = 29 \gamma B \]

35 \[ 24 \gamma B \quad +33 \gamma B = 57 \gamma B \]

40 \[ 55 \gamma B \quad +64 \gamma B = 119 \gamma B \]

Lin. increase \[ \text{for constant } \gamma \] \[ \text{taking } N_{q}/N_{q} = 1 \rightarrow \]

\[ q_{\text{ult}} \frac{d > 0}{d = 0} = 1 + \frac{2d}{B} = 3 \quad \text{for } d/B = 1/2 \]

Summary & Conclusions

1) Solution treats soil above footing as having weight only,
   i.e. NO STRENGTH (Hence \( \phi' \) of soil above footing is not relevant)

2) \( \phi' \), \( B \) and \( d/B \) all are VERY IMPORTANT

3) Should account for differing \( \gamma \) above/below footing

\* Actually not true since increasing \( B \) \( \Rightarrow \) increasing \( \sigma \) \( \Rightarrow \) decreasing \( \phi' \)
2.6 Effect of Soil Compressibility (Function of $D_r$)

(1) See attached Fig. 11.2 from Vasic (1973) on p8

- General Shear (high $D_r$) → well defined rupture surfaces and quiet ($D_r > 70\%$)
- Local Shear (medium $D_r$) → rupture surfaces beneath footing, but not outside $\phi$; quiet, not so clear
- Punching Shear (low $D_r$) → poorly defined; quiet
  with large settlements if don't mobilize shear in Zones II & III ($D_r < 35\%$)

(2) Empirical approaches used in practice

\* TIP (1967) - Loose sand: use $\tan \phi'_c = \frac{2}{3} \tan \phi'_p$

PHT (1974) - Attached Fig. 19.3 (p8) plots $N_p, N_q$ vs $\phi'$

$N_p / N_q$ (theory) = 0.7 to 0.9 with increasing $D_r$; $N_p \times N_q$ (theoretical)

- Vasic (1973) - Use $\tan \phi'_c = R.F. \tan \phi'_p$
  $R.F. = 0.67 + D_r - 0.75 D_r^2$ (for $D_r \leq 0.67$)

(3) Illustration

<table>
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<tr>
<th>$D_r (%)$</th>
<th>$\phi'$</th>
<th>$N_p$</th>
<th>$N_q$</th>
<th>$TIP (1967)$</th>
<th>$\phi'_c$</th>
<th>$N_p$</th>
<th>$N_q$</th>
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* Deleted because not in Terzaghi et al. (1996)
Modes of failure of model footings in Chattahoochee Sand (20,76)

Adapted from Vesic (1973) JSMFD, ASCE, 99(SMI)

Curves showing the relationship between bearing-capacity factors and $\phi$, as determined by theory, and rough empirical relationship between bearing capacity factors $N_r$ and $N_q$ and values of standard penetration resistance $N$.

Adapted from Peck, Hanson & Thornburn (1974)
### 2.7 Shape Factors (from Vesić, 1973) - Empirical factors from model footing tests

\[ Q_{ult} = S_c C N_c + S_y \frac{1}{2} \gamma B N_y + S_q \gamma d N_q \]

\[ S_c = 1 + \left( \frac{N_y}{N_c} \right) \left( \frac{B}{L} \right) \]

\[ S_y = 1 - 0.4 \left( \frac{B}{L} \right) \]

\[ S_q = 1 + \tan \phi \left( \frac{B}{L} \right) \]

For \( C = 0 \)

\[ Q_{ult} = \left( 1 - 0.4 \frac{B}{L} \right) \frac{1}{2} \gamma B N_y + (1 + \tan \phi \frac{B}{L}) \gamma d N_q \]

---

**Example**

\( \phi = 35^\circ, \gamma = 175 \text{ kN/m}^3 \)

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<tr>
<th>( d \text{ (m)} )</th>
<th>Shape</th>
<th>Component</th>
<th>( N_y )</th>
<th>( N_q )</th>
<th>( Q_{ult} (\text{kN/m}) )</th>
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<td></td>
<td>Square</td>
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<td>287</td>
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### 2.8 Inclined Eccentric Loadings (Strip)


\[ Q_{ult}(v) = \frac{Q_v}{B} = (1 - \frac{2e}{B})^2 \left( 1 - \frac{d}{\gamma} \right)^2 \frac{1}{2} \gamma B N_y \]

\[ + \left( 1 - \frac{2e}{B} \right) \left( 1 - \frac{d}{\gamma} \right)^2 \gamma d N_q \]

---

**Example from 2.7 for \( d = 3 \text{ m} \), \( \gamma = 10^\circ \text{ kN/m}^3 \), \( a = 0.1 \)**

\[ N_y \text{ Component: } (0.64)(0.5) = 0.32 \text{ kN} \times 210 \rightarrow 66 \]

\[ N_q \text{ Component: } (0.8)(0.79) = 0.63 \text{ kN} \times 175 \rightarrow 116 \]

\[ Q_{ult}(v) = 178 \times 0.66 ! \]
3. ESTIMATION OF Gt: IN PRACTICE (Footings on Sand)

3.1 Unit Weights (γ)

(1) Actual measurements
   - Test pits with in situ tests (bolloon, nuclear, etc)
   - Tube sampling — unless special procedures, disturbance 
     → Δγ (loose sand densifies; vice versa)

(2) Estimate from soil type & Dr. Some examples are:
   - LW Table 3.2
   - NAVFAC DM-7.1 (5/82) Fig 7 p71-149 (see sheet E)
   - But how estimate Dr? see section 3.3

(3) How important is error in γ?

\[ \gamma = 0.6 \pm 0.15 \]
\[ \text{Natural Sands} \]
\[ \text{For typical dry sand, } \gamma_d = 105 \pm 10 \text{pcf} \]
\[ \text{For submerged, } \gamma_b = 65 \pm 5 \text{pcf} \]
\[ \text{Error should be } < 10\% \]

\[ \text{Need values of } \gamma \text{ both above and below level of footing} \]
\[ \text{Need to estimate average } \gamma \text{ over } z = 0 \text{ to } \ell \]
3.2 Standard Penetration Test (SPT)

1) Test Procedures (ASTM D1586-84) Also see Sheet A

   - **Energy to Rod** ($E_R$)

   - **Anvil**
     - **Casing** (optional)
     - **Bottom of Hole**
     - **Rod**
     - **Split Spoon**

   - **$F_0 = \sigma_{ho} F_0$**

   - **$E_R = \sigma_{ho} F_0$**

   - **$E_R \times \sigma_{ho}$**

   - **$0-6$" $6$**
   - **$6-12$" $8$**
   - **$12-18$" $11$**

   - **$N = \text{blows} / 12 \text{ in.}$**

   - **Increasing embedment**

   - **Penetration resistance due to end bearing and friction**
     - For granular sands, $EB \times \sigma_{ho}$
     - For clays and silts, $F_0 \times \sigma_{ho}$ : $N$ increases with depth for homogeneous granular deposit

2) Factors Affecting $N$ (Other than depth & soil characteristics) See Sheet B

   - **Actual Energy ($E_R$) applied to top of rod = Energy Ratio ($ER$) x 350 ft-lb**
     - **Mainly weight of anvil**
     - **$ER = \text{Velocity Efficiency} \times \text{Dynamic Efficiency}$**
     - **Method used**
       - Automated release hammer
       - Repeatability

   - **$ER$ varies from $\approx 45\%$ for typical US practice with electric hammer & $2^\prime$ repetitive**
   - **to $\approx 80\%$ for Japanese practice with Tompi Tagger release**

   - **Recommended standard reference uses $ER = 60\%$**

   - **$N_{60} = N \times \frac{ER}{60}$**
b) Other factors include rod length, average ID of split spom and overage bore diameter, see Sheet B

3) Recommended standardized $N_{60}$: See Sheet B, Tables 617 & Table 5

$$N_{60} = CER \times CRL \times C_5 \times C_6 \times \text{Measured N}$$

- Oversize bore hole $(= 1.0 + 1.5)$
- US split spom W0 line $(= 1.2)$
- Length of rod $< 10 \text{m} (= 1.0 \times 0.75 \text{ at } x = 3-4 \text{m})$
- actual energy ratio $= \frac{ER}{60}$ $(= 1.3 \times \text{at } ER = 80\%$, typical Japan Tombi)
- Compared to 60%

**Table 5 of Skempton (1984) Geot. 36(3), 425-447**

<table>
<thead>
<tr>
<th>Release Type</th>
<th>Cathead</th>
<th>VE (%)</th>
<th>Hammer System</th>
<th>Anvil weight: kg</th>
<th>DE</th>
<th>ER (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Waterways Experiment Station</td>
<td>Trip</td>
<td>—</td>
<td>100</td>
<td>Vicksburg</td>
<td>0</td>
<td>0.83</td>
</tr>
<tr>
<td>Japan</td>
<td>Tombi</td>
<td>—</td>
<td>100</td>
<td>Donut</td>
<td>2</td>
<td>0.78</td>
</tr>
<tr>
<td>USA</td>
<td>Slip-rope (2 turns) Small</td>
<td>83</td>
<td></td>
<td>Donut</td>
<td>2</td>
<td>0.78</td>
</tr>
<tr>
<td>UK</td>
<td>Slip-rope (1 turn) Small</td>
<td>85</td>
<td></td>
<td>Safety</td>
<td>2.5</td>
<td>0.79</td>
</tr>
<tr>
<td>USA</td>
<td>Slip-rope (2 turns) Large</td>
<td>70</td>
<td></td>
<td>Old standard</td>
<td>3</td>
<td>0.71</td>
</tr>
<tr>
<td>UK</td>
<td>Trip</td>
<td>—</td>
<td>100</td>
<td>Donut</td>
<td>12</td>
<td>0.64</td>
</tr>
<tr>
<td>USA</td>
<td>Slip-rope (2 turns) Large</td>
<td>70</td>
<td></td>
<td>Pilcon</td>
<td>19</td>
<td>0.60</td>
</tr>
<tr>
<td>USA</td>
<td>Slip-rope (2 turns) Trip</td>
<td>—</td>
<td>100</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

VE = Velocity Efficiency  DE = Dynamic Efficiency  ER = Energy Ratio

Some examples of reported $N$ for $N_{60} = 20$ ($l > 10 \text{m}$, dia $\leq 4.5")$

1) USA, donut with 12 kg anvil, 2 rope turns on large cathead
   - Old data with old samplers (ID = 35 mm) $N = 20/(45/60) = 26.7 \approx 27$
   - New sampler without line $N = 20/(45/60 \times 1.2) = 222 \approx 22$

2) Japan, donut with 2 kg anvil
   - Tombi release $N = 20/(78/60) = 15.4 \approx 15$
   - 2 rope turns, small cathead $N = 20/(65/60) = 18.5$

(1) USA, donut with 12 kg anvil, 2 rope turns on large cathead
   - Old data with old samplers (ID = 35 mm) $N = 20/(45/60) = 26.7 \approx 27$
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(2) Japan, donut with 2 kg anvil
   - Tombi release $N = 20/(78/60) = 15.4 \approx 15$
   - 2 rope turns, small cathead $N = 20/(65/60) = 18.5$
4) Effect of Depth and OCR (At constant Dr)

a) General
- Increasing $Z \rightarrow$ increasing $\sigma' \rightarrow$ increasing $EB/\sigma'
- OCR $\Rightarrow \sigma'_{ho} \rightarrow F$
- At constant OCR $\frac{N}{D^2} = a + b \sigma'_{vo}$

b) Sources of data
- California Chambre: USBR & Gibb & Holtz (1957) Coarse silt & fine sand (bird tests)
- WES: Marcham & Beganashy (1977) Coarse, medium, fine sand
- Field data: Bek & Began (1969) University of London, dense coarse sand
- Skempton (1986) Added data from Japan

c) Objective: Develop a relationship to obtain a corrected $N$ at a reference overburden stress (Most use $\sigma'_{vo} = 175\text{ SF} = 1 \text{ kg/cm}^2 \approx 105\text{ kPa}$) $N = C_N N$. See Sheet C for equations & comparisons.

d) CCL Recommendation to obtain $N = C_N N$
- For $\sigma'_{vo} > 1\text{ atm}$
  - Leaird & Whitman (1986)
  - $C_N = \sqrt{\frac{1}{\sigma'_{vo} \text{ (SF)}}} = \frac{10}{\sqrt{\sigma'_{vo} \text{ (kPa)}}}$
  - (Simple to remember and placed in middle, but)
  - Gives $C_N$ too high at $\sigma'_{vo} < 1\text{ atm}$
- For $\sigma'_{vo} < 1\text{ atm}$
  - Skempton (1986)
  - $C_N = \frac{2}{1 + \sigma'_{vo} \text{ (SF)}} = \frac{2}{1 + 0.01 \sigma'_{vo} \text{ (kPa)}}$
  - (Fairly simple and placed in middle)

- Values: $\sigma'_{vo} \text{ (SF)} = 0.25 \ 0.5 \ 1.0 \ 1.5 \ 2.0 \ 3.0$
- $C_N = 1.60 \ 1.33 \ 1.0 \ 0.82 - 0.71 \ 0.58$
3.3 Estimation of Dr From SPT N Data

1) Historical Perspective

a) Proposed correlations (modified for this summary); $N_1 = \text{corrected } N_1 \text{ at } D_{50} = 1 \text{ cm}$

(1) Peck & Baggeroar (1969) JSM/FO 95 (59): Field data on dense, coarse (OC) sand

\[ D_r = \sqrt{\frac{N_1}{85}} \quad \text{à la Skempton (1966)} \]

"Old" US practice → very low ER (high $N_1$)

(2) Helly & Geikie (1979) JGEO 105(3): Mean from late to on coarse to fine sand

\[ D_r = \sqrt{\frac{N}{16 + 23 \sigma_i^2 (\text{SF})}} \quad \rightarrow \quad \sqrt{\frac{N_1}{39}} \]

Probably high ER (low $N_1$)

(3) Marcuson & Brandanou (1971) JGEO 103 (76, 11): Mean from late to on fine to coarse to fine sands

\[ D_r (\% ) = 12.2 + 0.75 \left( \sqrt{222N + 2311 - 711(\text{OCR}) - 736 \sigma_i^2 (\text{SF})} - 50 \sigma_i^2 \right) \]

* For OCR = 1, Skempton developed:

\[ D_r = \sqrt{\frac{N_1}{52 \rightarrow 33}} \]

Very high ER (low $N_1$)

\[ D_{50} = 2 \text{ mm} \]

b) Comparison of correlations

(see also p.15)

<table>
<thead>
<tr>
<th>$N_1$</th>
<th>P&amp;B (69)</th>
<th>H&amp;G (76)</th>
<th>M&amp;B (77)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>34</td>
<td>51</td>
<td>44 → 55</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>C → F sand</td>
</tr>
<tr>
<td>30</td>
<td>59</td>
<td>88</td>
<td>76 → 95</td>
</tr>
<tr>
<td></td>
<td>FIELD</td>
<td>LAB</td>
<td>C → F sand</td>
</tr>
</tbody>
</table>

c) Why so different (largely from Skempton 1980)?

(1) Large difference in Energy Ratio (ER), e.g., $(N_1)_{60} = 0.75 N_1$ in P&B (69) with ER = 45% vs. $(N_1)_{60} = 1.1 N_1$ in M&B (77)

(2) Lab testing on freshly deposited sands vs. field data on "aged" deposits, plus also maybe OC (both → increased $N_1$ at same $D_r$)

(3) At same $D_r$, increasing grain size ($D_{50}$) → increasing $N_1$
Fig. 47. Empirical correlations between standard penetration resistance and relative density for cohesionless soils.

Adapted from Ladd et al. (1977) 9th ICFMFE
2) Results from Skempton (1968)

- He evaluated above historical data plus Terzaghi's Peak (1948-1967) plus field data from Japan in terms of \( N_{160} \), i.e. accounted for variations in energy ratio and \( N \) corrected to \( N_{160} = 160 \).
- Sheet D summarizes Skempton's results, which led to conclusions in 1c.
- Note that TIP correlation can be closely fitted by \( D_r = \sqrt{\frac{N_{160}}{60}} \)

3) CCL Recommended Correlations

a) Natural sand deposits

\[ D_r = \sqrt{\frac{N_{160}}{55 \text{ fine sand} \ 65 \text{ coarse sand}}} \]

Note: May overestimate \( D_r \) for high OCR sands.

b) New fill deposits

\[ D_r = \sqrt{\frac{N_{160}}{55 + 25 \log D_{50} (\text{mm})}} \]

c) See Sections 3.2-3.4 for procedures to estimate \( N_{60} \) \( \# \) \( N_{160} \) respectively.

For typical US practice using 2·ope turns on large cathedal:

- Donut hammer, standard sampler: \( N_{60} = 0.75N \)
- " " " normal \( q=38 \text{mm} \): \( N_{60} = 0.9N \)
- Safety hammer, " " " : \( N_{60} = 1.1N \)

\[ \text{For } l > 10 \text{m (30')} \]

\[ \text{See Sheet B, Table 7} \]

\[ \text{for } l < 10 \text{m} \]
3.4 Estimation of $\phi'$ From Lab Testing

1) On "undisturbed" samples: 2 problems
   - Change in density during sampling à la Section 3.1.1
   - Very difficult & expensive to set up test specimen

2) On reconstituted samples: 2 problems
   - Potential error in estimating $D_r$ (Section 3.2), plus need $e_{max} - e_{min}$
   - Preparation technique to simulate natural sand structure

3) Conclusion: On very important jobs, consider using procectent freezing and sampling → lab testing

3.5 Estimation of $\phi'$ From $D_r$ and Sand Type

1) Very indirect method, e.g., estimating $D_r$ (usually from $N$ data) and then estimating $\phi'$ vs. $D_r$ as function of sand type (USCS)

2) Sheet E contains two $\phi'_{p}$ vs $D_r$ correlations
   - DM-7.1 probably is rather conservative $\rightarrow \phi' = 36 \pm 10$
   - Schmuitjen (1978) probably is upper limit $\rightarrow \phi' = 40 \pm 2$

3) CCL also would use Belton (1986), although this approach requires an estimate of $\phi'_{CS}$ (rule Table 1 $\rightarrow \phi'_{CS} = 38 \pm 2.5$ for mostly uniform fine to coarse sands)

3.6 Estimation of $\phi'$ From SPT $N$ Data

1) Fig. 1 in Sheet F presents two early correlations. Note significant difference in $\phi'$ at same $N \geq 10$. When using this chart, CCL recommends correcting the measured $N$ to $\sigma'_{vo} = 1$ atm, i.e., use $N_{r}$ (or even $(N)_{60}$).

For SPT in fine and sandy sands, Meyerhof (1956) recommended reduced $N' = N + 0.5(N - 15)$ for $N > 15$ (due to partial drainage of dilatant sand, but only if below the water table)
2) Fig. 2 in Sheet F presents correlations between $\phi'$ and $N_1$ from TPM’s (996) book based on "various proposals" (PHT ‘53, O’Neill ‘68, Schmertmann ‘75 & Stroud ‘85). Further comments are:
- Underestimates $\phi'$ for calcareous sands (due to particle crushing)
- Overestimates $\phi'$ for OC sands (due to increased $K_0$-more side friction)
- Agrees reasonably well with CPT correlations (see Section 3.8) that used a different database

3.7 Estimation of $\phi'$ from Plate Load Tests (PLT)

1) PLT procedure à la ASTM D1190

   - $P/A = 6-30$ in
   - Maintain each load until $dP/dt < 0.001$ in/min for $\Delta t = 4$ min
   - Std. Load Test (TIP, 1962)

   2) $\phi'_{ult} = (0.6) \frac{1}{2} VIB N_{80}$
   - Estimate $\phi'$ from measured $V_{IB}$.

   3) Remarks: very expensive and must test soil at representative depth

3.8 Correlations for Cone Penetration Tests (CPT)

1) CPT procedure à la ASTM D3441-94 (For Electric CPT)

   - Penetrate at 1-2 cm/sec
   - Internal load cell measures $q_c$

   $q_t = q_c + U(1-a)$

   Note: Really should measure $q_t = q_c + U(1-a)$

   Where $a = \frac{D^2}{d^2}$ (usually 0.5 to 0.85, but)

   $D = 10 \text{cm}^2$
2) Relationship between CPT \( q_c \) and SPT \( N \)
   
   See Sheet G, Fig. 11.57: \( q_c(bas)/N = 4 \text{ to } 8 \text{ for fine to coarse sand} \)

3) Estimation of \( D_r \) (Sheet G, mostly using data set leading to Sheet H)
   
   Fig. 4 shows that sand compressibility affects \( D_r = f(q_c, \sigma_0') \),
   
   i.e., higher compressibility \( \Rightarrow \) lower \( q_c \) at same \( D_r, \sigma_0' \)

   Fig. 5 presents \( D_r = f(q_c, \sigma_0') \) for NC sands of moderate compressibility
   
   Authors suggest using \( \sigma_0' \) for OC sands; they also state that Fig. 5 is "approximate and should be used as a guide" due to unknown sand compressibility at high \( \sigma_0' \) levels around cone tip

4) Estimation of \( \phi' \)
   
   a) Sheet H presents evaluation of data from several series of
      
      Katz in calibration (bin) chambers

      Fig. 6 - Compares \( q_c/\sigma_0' \) in theory and experimental data.

      Fig. 7 - Proposed correlation between \( q_c \) vs \( \sigma_0' \) and \( \phi' \), where
      
      \( \phi' = \phi_0' \text{ for TC with } \sigma_0' = \sigma_c = \text{ maximum } \sigma_0' \text{ for NC quartz sands} \)

      Note linear \( q_c = \sigma_0' \) which is surprising to CCL

   b) Sheet I presents correlation in TPM ('96) that presumably used
      
      same data set as for Sheet H, but now plotted as \( q_c \), assuming
      
      \( q_c(kPa) = 10 \sqrt{\sigma_0'} \text{ kPa} \) (i.e., same eqn. used to get \( N_1 = C_N N_2 \text{ in p13} \)

      CCL added data scaled from Fig. 7 (Sheet H) at \( \sigma_0' = 16 \text{bar} = 160 \text{kPa} \)

      CCL also added eqn. for \( \phi' = f(q_c) \), which may be valid only for \( q_c \) data obtained at \( \sigma_0' \approx 1 \text{ atm} \)
Figure A-4. Equipment Used to Perform the SPT

Source: Kovacs, et al.

Kulhawy & Mayne (1990) Cornell Report to EPR2

ASTM D 1586-84 Note that ID = 1.5" (38mm) enables use of a thin liner to end up with an ID = 1 3/8" (35mm), which is the original dia. considered an international standard. However, liner is seldom used in the US!

Information on SPT Equipment
A. Energy Efficiency

1) Energy delivered to rod stem, \( E_R = ER \times (W \times h = 140 \times 6x \times \frac{30}{12} h = 350 h/16) \)

2) Factors affecting Energy Ratio (\( ER \)) = Velocity Effici. (\( VE \)) \times Dynamic Effici. (\( DE \))

   a) \( VE \) mainly affected by release mechanism:
      - Automated (hinge/top) \( \rightarrow VE = 1.0 \)
      - Rope around cathead \( \rightarrow VE = 0.7 - 0.85 \) for 2 turns

   b) \( DE \) affected by weight of anvil: \( m_{an} \) (kg) \( \rightarrow \) den. \( DE \) (0.8 - 0.8)

3) \( ER = 60\% \) accepted as best reference, \( N_{60} \approx N (ER/60) \)

B. Other Factors (Table 7 & Fig. 5)

1) Rod length < 10 m \( \rightarrow \) higher \( N \)
2) No. hammer \( \rightarrow \) lower \( N \)
3) Large boring depth \( \rightarrow \) lower \( N \) (granular)

C) \( N_{60} \) Eq. \( N_{60} = CER \cdot CR \cdot CS \cdot CB \cdot \text{measured} \ N \)

---

**Table 6. Summary of rod energy ratios** (Skempton 1986)

<table>
<thead>
<tr>
<th>Hammer</th>
<th>Release</th>
<th>( ER_% )</th>
<th>( ER/60 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Japan</td>
<td>Donut</td>
<td>78</td>
<td>1.3</td>
</tr>
<tr>
<td></td>
<td>Donut</td>
<td>65</td>
<td>1.1</td>
</tr>
<tr>
<td>China</td>
<td>Picon type, Donut</td>
<td>60</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>Trip, Manual</td>
<td>55</td>
<td>0.9</td>
</tr>
<tr>
<td>USA</td>
<td>Safety Donut</td>
<td>55</td>
<td>0.9</td>
</tr>
<tr>
<td></td>
<td>2 turns of rope</td>
<td>45</td>
<td>0.75</td>
</tr>
<tr>
<td>UK</td>
<td>Picon, Dando, old standard</td>
<td>60</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>2 turns of rope</td>
<td>50</td>
<td>0.8</td>
</tr>
</tbody>
</table>

---

**Table 7. Approximate corrections to measured \( N \) values** (Skempton 1986)

<table>
<thead>
<tr>
<th>Rod length:</th>
<th>C_{RL}</th>
<th>C_{S}</th>
<th>C_{B}</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; 10 m</td>
<td>1.0</td>
<td>0.95</td>
<td>0.75</td>
</tr>
<tr>
<td>6-10 m</td>
<td>1.2</td>
<td>0.85</td>
<td></td>
</tr>
<tr>
<td>3-4 m</td>
<td>1.0</td>
<td>0.85</td>
<td></td>
</tr>
<tr>
<td>Standard sampler</td>
<td></td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>US sampler without liners</td>
<td></td>
<td>1.2</td>
<td></td>
</tr>
<tr>
<td>Borehole diameter: 65-115 mm</td>
<td>1.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>150 mm</td>
<td>1.05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>200 mm</td>
<td>1.15</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

---

**Figure 2-17. Energy Ratio Variations**

**Factors Affecting SPT \( N \) Values** (Skempton 1986)
A. Equations for $C_N$:

1) Peck, Hanson & Thornburn (1974) Book:
   Evaluation of Peck & Bazaraa (1969) field data
   \[ C_N = 0.77 \log \left( \frac{20}{\sigma_v'(TSF)} \right) \]

2) Seed (1976) ASCE Report:
   Evaluation of Gabla & Holty (1957) lab data
   \[ C_N = 1 - 1.125 \log \sigma_v'(TSF) \]

3) Seed (1979) JCE 105(2):
   Evaluation of Marcuson et al. (1977) lab data
   \[ C_N = \frac{2}{1 + \sigma_v'(TSF)} \] (for $D_r = 50\%$ and $D_r = 70\%$)
   (no equation)

4) Liao & Whitman (1986) JCE 112(3):
   Evaluation of prior correlations
   \[ C_N = \frac{3}{2 + \sigma_v'(TSF)} \] (for fine sands of medium $D_r$)
   \[ C_N = \frac{3}{2 + \sigma_v'(TSF)} \] (for dense coarse sands under NC)

B. Comparisons

[Graph showing SPT Overburden Correction Factor, $C_N$ vs. $\sigma_v'_{(TSF)}$]

- Peck, et al., 1974 (34)
- Liao and Whitman, 1986 (33)
- Skempton, 1966 (31):
  - Coarse sands
  - Fine sands
  - O.C. sands
- Seed, 1979 (35):
  - $D_r = 50\%$
- Seed, 1979 (35):
  - $D_r = 70\%$

[Graph showing $\sqrt{1/\sigma_v'}$ vs. $C_N$]

- Skempton (1986)
- Liao & Whitman (1987)
  - Geot. 37(3)

Kulhawy & Mayne (1990)
From Skempton (1986: Geoh 36(3)) \( (N_{160}) \) measured \( N \) corrected to Energy Ratio of 60% and \( \sigma'_{vo} = 1 \text{ atm} = 1.02 \text{ kN/m}^2 = 100 \text{ kPa} \)

**Skempton interpretation of Terzaghi & Peck (1948)**

For \( 35\% < D_r < 85\% \), \( OCR = 1 \), and \( 0.5 < \sigma'_{vo} < 2.5 \text{ atm} \), closely fitted by

\[
D_r = \sqrt[3]{\frac{(N_{160})}{60}}
\]

**NOTE:** For some \( D_r \), \( (N_{160}) \) increases with:

1. Increasing mean grain size, \( D_{50} \)
2. Aging. Therefore higher for natural deposits than for recent fills and lab testing programs.
3. Overconsolidation ratio, \( OCR \)
Angle of Internal Friction Vs Density (For Coarse Grained Soils)

\[ \phi' \] Obtained from effective stress failure envelopes approximate correlation is for cohesionless materials without plastic fines

- **ML**
- **SM** and **SP** in line range
- **SW**
- **GP**
- **GW**

**Correlations Of Strength Characteristics For Granular Soils**

Adapted from NAVFAC DM-7.1 (5/82) p 7.1 - 149

Chart for the approximate evaluation of the peak angle of internal friction after the relative density has been evaluated. Modified from: Burmister, Donald M., “The Importance and Practical Use of Relative Density in Soil Mechanics,” ASTM Proc., Vol. 48, 1948.
Fig. 1 "Early" Correlations

Fig. 2 Recent Correlation

Friction Angle from SPT Blowcount: Empirical Correlations
Figure 11.15  Relation between cone penetration resistance $q_c$ and standard penetration $N$ or $N_{60}$ values of sands as related to the median particle size $D_{50}$ of the sands.

(Terzaghi, Pack & Mesri 1968)

Correlations Between SPT N Values and CPT-Cone Resistance
Fig. 6. Relationship between bearing capacity number and peak friction angle from large calibration chamber tests.

Robertson & Campanella (1983)
CGJ 20(4)

NOTE: $\phi' = \phi'_{rc}$ for $\sigma'_3 = \text{In-Situ} \sigma'_{ho}$; also note linear: $q_c$ vs $\sigma'_v$ relationship
Figure 19.5  Empirical correlation between friction angle $\phi'$ of sands and normalized push cone tip penetration resistance. (Terzaghi, Pach & Mesri 1996)

Scaled from Fig. 1 (Sheet H) at $\sigma'_{vo} = 1$ bar; therefore $q_c = q_{ci}$

Linear regression $\rightarrow \phi' = 30.0 \left( \frac{q_{ci}}{\sigma'_{vo}} \right)^{0.136} \quad (n = 9, r^2 = 0.986)$

Correlation line on Fig. 19.5 $\rightarrow \phi' = 28.8 \left( \frac{q_{ci}}{\sigma'_{vo}} \right)^{0.145}$; however, Fig. 1 correlation shows $q_c \propto \sigma'_{vo}$, not $q_c \propto 1/\sqrt{\sigma'_{vo}}$ as assumed in Fig. 19.5