Cavity collapse

A spherical cavity of initial radius $R_o$ suddenly collapses. Ignore surface tension and water compressibility. The cavity has the pressure $p_o$ which is smaller than the pressure at infinity $p_\infty$.

Use continuity that

$$4\pi r^2 u = \text{constant for } r \geq R(t)$$

where $R(t)$ is the radius of the cavity, and $u(r,t)$ is the radial velocity. Use also the law of momentum conservation

$$\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} \right) = -\frac{\partial p}{\partial r}$$

Integrate the momentum equation from $r = R$ to $r = \infty$ to get a differential equation of the following form

$$R \frac{d^2 R}{dt^2} + \frac{3}{2} \left( \frac{dR}{dt} \right) = -\frac{p_\infty}{\rho}$$

and show that it can be rewritten as

$$\frac{1}{2} \frac{d}{dt} \left( R^2 \left( \frac{dR}{dt} \right)^2 \right) = -\frac{\Delta p}{3\rho} \frac{d(R^3)}{dt}$$

With the initial conditions

$$R(0) = R_o, \quad \frac{dR(0)}{dt} = 0$$

Show that the time to complete collapse is

$$T = \sqrt{\frac{3\rho}{2\Delta p}} \int_0^{R_o} \frac{dR}{\sqrt{\frac{R}{R_o} - 1}}$$

Evaluate the integral in terms of Gamma functions.