1. Oil slick on the sea surface

Let the oil slick be modelled by an inextensible, massless and perfectly flexible film floating on the sea surface. There is no wind above but a surface wave passes beneath causing the surface and the film to have the following vertical displacement

\[ \zeta = A \sin(kx - \omega t) \]  \hspace{1cm} (1)

with \( kA \ll 1 \) and \( \omega = \sqrt{gk} \) being the dispersion relation for deep water waves. Waves introduce a Stokes boundary layer under the film, and Eulerian streaming in the boundary layer, causing the film to drift at some mean velocity \( U \). Find the drift velocity \( U \) by a perturbation analysis.

Suggestions:

1. Consider the water motion induced by an inextensible film having \( \zeta = (A \sin(kx - \omega t)) \) and make use of the following result that a point on the film has the velocity components to the leading order

\[ u(x, \zeta, t) = \frac{A^2}{4} \omega k \cos(2kx - \omega t) \left[ 1 + O(kA)^2 \right], \]  \hspace{1cm} (2)

\[ v(x, \zeta, t) = -\omega A \cos(kx - \omega t) \left[ 1 + O(kA)^2 \right] \]  \hspace{1cm} (3)

These results are due to G.I. Taylor for another problem.

2. Assume for simplicity that the amplitude \( A \) is much smaller than the boundary layer thickness \( \delta = \sqrt{2\nu/\omega} \).

3. Find the second-order Eulerian streaming velocity in the boundary layer and its limit \( \bar{u}_\infty \) at the outer edge of the boundary layer.

4. The film must advance at the drift velocity \( U = -\bar{u}_\infty \).