

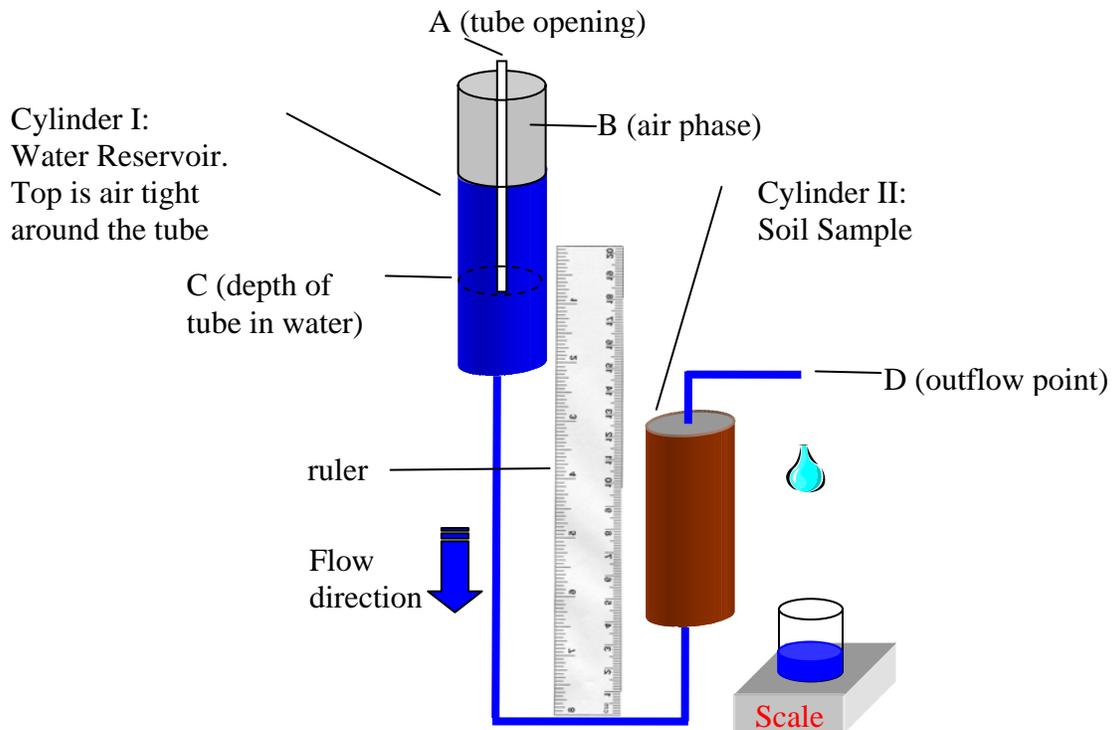
Solutions

The purpose of this assignment is to familiarize you with the field equipment you will use at Waquoit Bay (WB) on the field trip Sunday. At WB, you will attempt to demonstrate Darcy's Law:

$$q = K \frac{dh}{dz} \quad (1)$$

by independently measure its three components: K , the hydraulic conductivity of the bay sediment; q , the Darcy flux of water moving from the subsurface into the bay ; and $\frac{dh}{dz}$, the hydraulic gradient between the subsurface and the bay. You will measure each component by the following means: hydraulic conductivity with a permeameter; the Darcy flux with a seepage meter, and the hydraulic gradient with a differential head sampler. The purpose of this exercise is to get familiar with and understand the equipment you will use. Please study the figures and reason through the following questions.

(20/20) Hydraulic Conductivity: The Permeameter



(5/5) The purpose of Cylinder I is to maintain a constant head. In Cylinder I, what happens in part **B** as water starts to flow?

When water flows, the loss of water volume causes the air phase to expand, which decreases its pressure below atmospheric.

(5/5) How does this cause part **A** and **C** to respond?

The decrease in pressure at B causes a head gradient in the air phase between tube A and part B, and this causes air bubbles to flow from the bottom of tube A, through the water phase and up into part B. Basically the tube bubbles to balance the pressure.

(5/5) How does this affect the hydraulic head level at part **C** as water continues to flow?

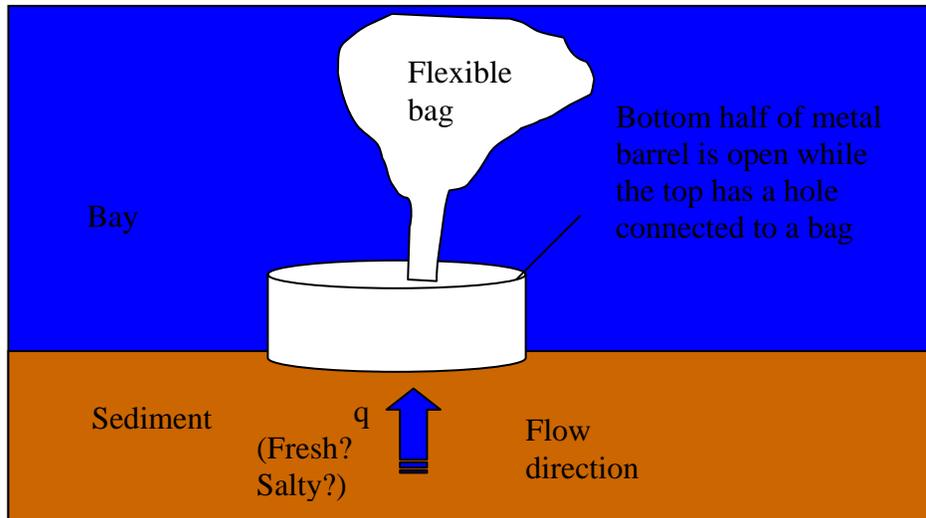
Air will flow in to replace the water that is leaving. The bubbling action means that the air-water interface within the system is at the bottom of tube A (or at point C). Thus at point C the pressure is atmospheric. (The less-than-atmospheric pressure in B plus the weight of the water above point C has to equal atmospheric pressure) As a result, the hydraulic head is held constant at the bottom of the straw. (Think of the head as the water level in the straw, which is at point C, because the tube is the only part open to the atmosphere. The water above point C is held up by a less-than-atmospheric pressure in part B).

(5/5) Knowing that fresh water is 1 g/ml and having a stop watch handy to record the weight of water accumulation from the scale with time, what other easy measurement(s) are required to apply Darcy's Law in order to determine the hydraulic conductivity of the soil sample in cylinder II? How can the permeameter measure the hydraulic conductivity of a soil sample?

You need to know the two head levels which are: H-1) the bottom of the straw and H-2) the outflow port. You also need to know the cross sectional area of cylinder 2, A and the length of the cylinder, L. Then by recording the volume of water flowing out with time, Q, you can apply Darcy's Law to determine K:

$$q = -K \frac{dh}{dz} \Rightarrow \frac{Q}{A} = K \frac{H_1 - H_2}{L} \Rightarrow$$
$$K = \left(\frac{Q}{A} \right) / \left(\frac{H_1 - H_2}{L} \right)$$

(15/15) Darcy flux: The Seepage Meter



(7/7) Using a scale to weigh the bag after a recorded amount of time what other measurements are needed to measure the Darcy flux? Do you think it would be important to fill the bag with some water first? Why?

To get the flux we need to know the area of the seepage meter because flux is flow divided by area. (The flow is determined by dividing the volume of water collected (or lost) from the bag by the time that the seepage meter has been operating. The flux is then determined by dividing the flow by the area of the seepage meter.) Additionally, since we are converting mass to volume we need measurements of the salinity and water temperature so we can calculate an accurate density.

It is important to fill the bag with water first incase the flux of water is in the other direction than what we assume (i.e. we have observed flow from the bay into the sediment rather). We would still like to know what the flux is even if it is in the other direction.

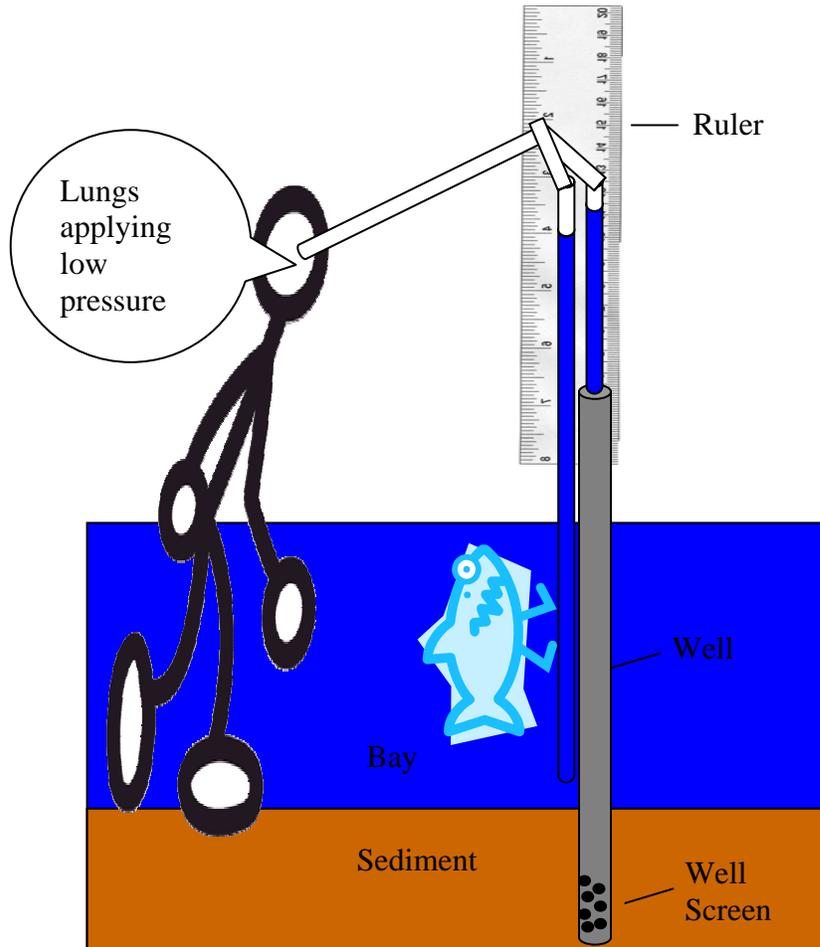
(8/8) Why doesn't the Bay squeeze all the water out of the bag? How is this device used to directly measure the Darcy flux?

The bay does not squeeze water out of the bag because the bag and the bay have the same hydraulic head. At a given height, the water in the bag experiences the same hydrostatic pressure as the water in the bay. [Head = $Z + (p-p_o)/(\rho g)$].

The seepage meter directly measures the Darcy flux by capturing the water that moves in or out of the sediment. The key to the seepage meter is that the head in the seepage meter

bag is the same as the bay water and the head in the sediment is the same in the bay and under the seepage meter. This allows us to accurately measure natural flow conditions.

(15/15) Hydraulic Gradient: The Differential Head Sampler



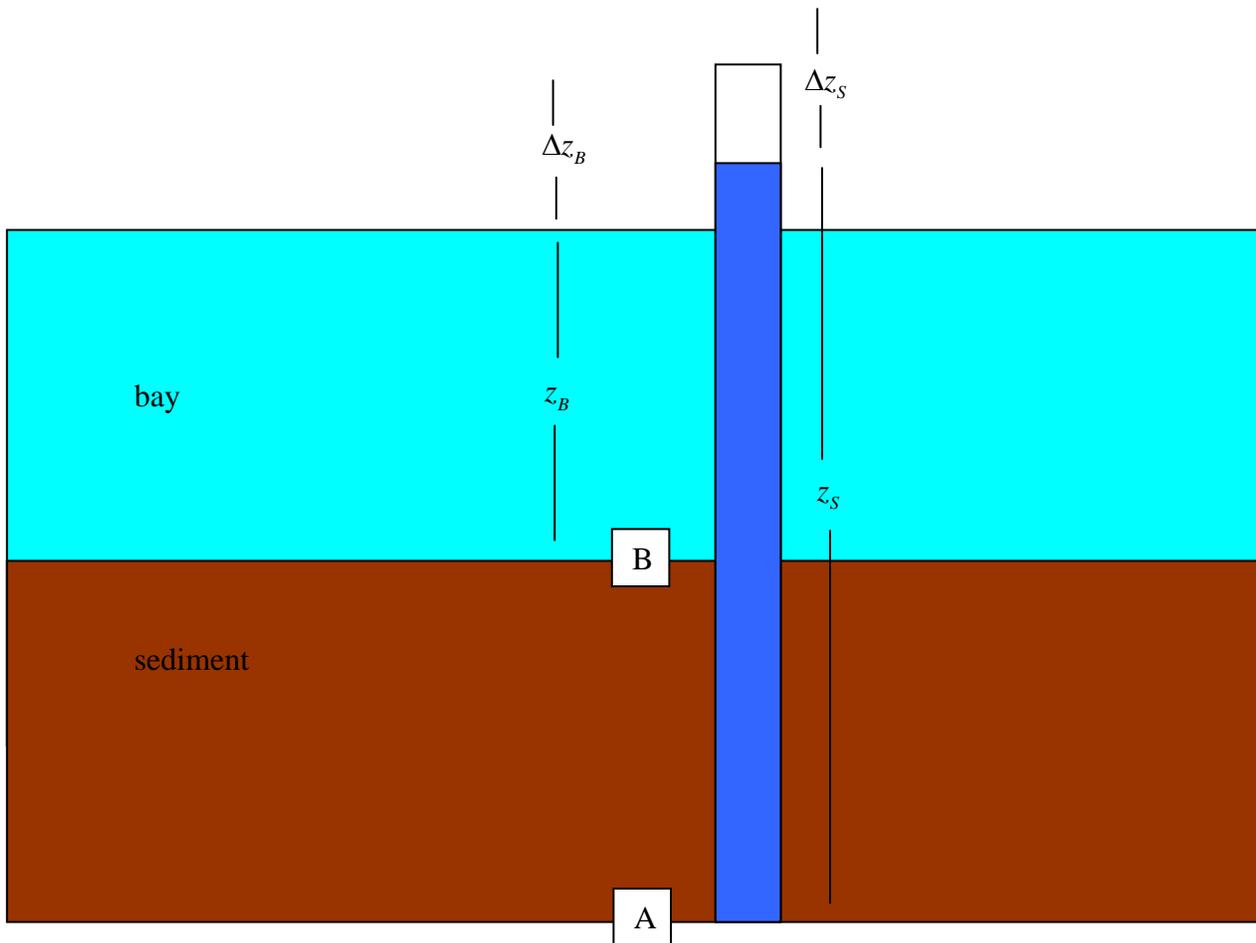
(8/8) Aside from drowning, why don't we care how deep the bay is (which is why the ruler stays above water)? Do we care how deep the well goes into the sediment? How can this device be used to calculate the hydraulic gradient?

We don't care how deep the well goes into the sediment because we are interested in the hydraulic gradient (dh/dz). " dh " is the head difference between the top of the sediment and the location of the well screen. " dz " is the depth of the well in the sediment. We assume that the hydraulic gradient is constant along the sediment profile, such that if we put the well in deeper (a larger " dz ") we will measure a larger " dh ".

This device provides the hydraulic gradient because the water in the well (and the water in the tube in the well) has the same head as the water around the well screen, and the water in the tube in the bay has the same head as the bay. When a low pressure is applied to the tubes it causes the water in both tubes to rise an equal amount (assuming density of the water in the two tubes is the same). The difference in height between the two water columns is the difference in head, just translated up to a point where we can visualize the difference.

(7/7) Lets say the bay water is saltier than the sediment water, how does this affect the gradient? Can we correct for it?

We want to calculate the hydraulic head difference at points A and B, because that's what drives flow through the sediments.



The difficult aspect of this problem is the bay and sediment water can have different salinities and hence densities, which makes measuring the true head gradient problematic (not to mention makes it necessary to modify hydraulic conductivity calculated in lab

with freshwater. Think about the relationship between hydraulic conductivity and intrinsic permeability). First we need to convert bay hydraulic heads into equivalent sediment hydraulic heads (because we are interested in water flow in the sediments) to make the head differences comparable. Remember that hydraulic head is the energy normalized to the unit weight of the fluid. To compare two fluids with different densities, it is necessary to convert one hydraulic head to an equivalent hydraulic head with the same unit weight. From a force balance, we can calculate how to convert hydraulic bay head into an equivalent hydraulic sediment head:

$$\rho_S z_{S,E} g = \rho_B z_B g \Rightarrow z_{S,E} = \frac{\rho_B}{\rho_S} z_B \quad (1)$$

where ρ_s is the density of sediment water, ρ_b is the density of bay water, z_b is the height of the bay water (or the head of the bay water), $z_{s,e}$ is the height/head of the bay water if it were to have the same density of the sediment water – it is the equivalent height of sediment water.

This means the true hydraulic head difference across the sediment that we wish to measure is:

$$\Delta h_s = A - B = z_s - z_{s,E} = z_s - \frac{\rho_B}{\rho_S} z_B \quad (2)$$

When we apply pressure to the differential head sampler (which we did with our lungs at Waquoit Bay) the sediment and bay water will rise different distances Δz 's because of their different densities. Again applying a force balance we know:

$$\Delta z_B g \rho_B = \Delta z_s g \rho_s = P_{\text{applied}} \quad (3)$$

The difference that we actually measure with the differential head sampler is:

$$\Delta h_{s, \text{measured}} = (z_s + \Delta z_s) - (z_B + \Delta z_B) \quad (4)$$

Solving for Δz_s from 3, and plugging in to four yields:

$$\Delta h_{s, \text{measured}} = \left(z_s + \Delta z_B \frac{\rho_B}{\rho_S} \right) - (z_B + \Delta z_B) = \quad (5)$$

$$\Delta h_{s, \text{measured}} = (z_s - z_B) + \Delta z_B \left(\frac{\rho_B}{\rho_S} - 1 \right)$$

But what we really want is:

$$\Delta h_s = z_s - \frac{\rho_B}{\rho_s} z_B \quad (6)$$

Note that the true head difference equals the measured only when the densities are equal. This means to calculate the true head gradient that we want, we would have to also measure the depth of the bay water, the height that the bay water is pulled above the bay when we apply a pressure to the differential head sampler, and the two densities (salinities). Then we could solve for z_s and plug into equation 6.

For a typical case at WB, the flux is 0.1 m/day and the hydraulic conductivity is around 25 m/day, which would mean that the gradient would have to be $.1/25 = 0.004$. Driving in the peizometer 75 cm means the true hydraulic head drop would be: 0.3 cm
 Lets also assume that the bay depth is 75 cm and the bay density is 1025 grams/liter.

Apply equation 6, we can calculate what the head in the sediments should be as a function of its salinity:

$$0.3 = z_s - \frac{1025}{\rho_s} * 75 \Rightarrow \quad (7)$$

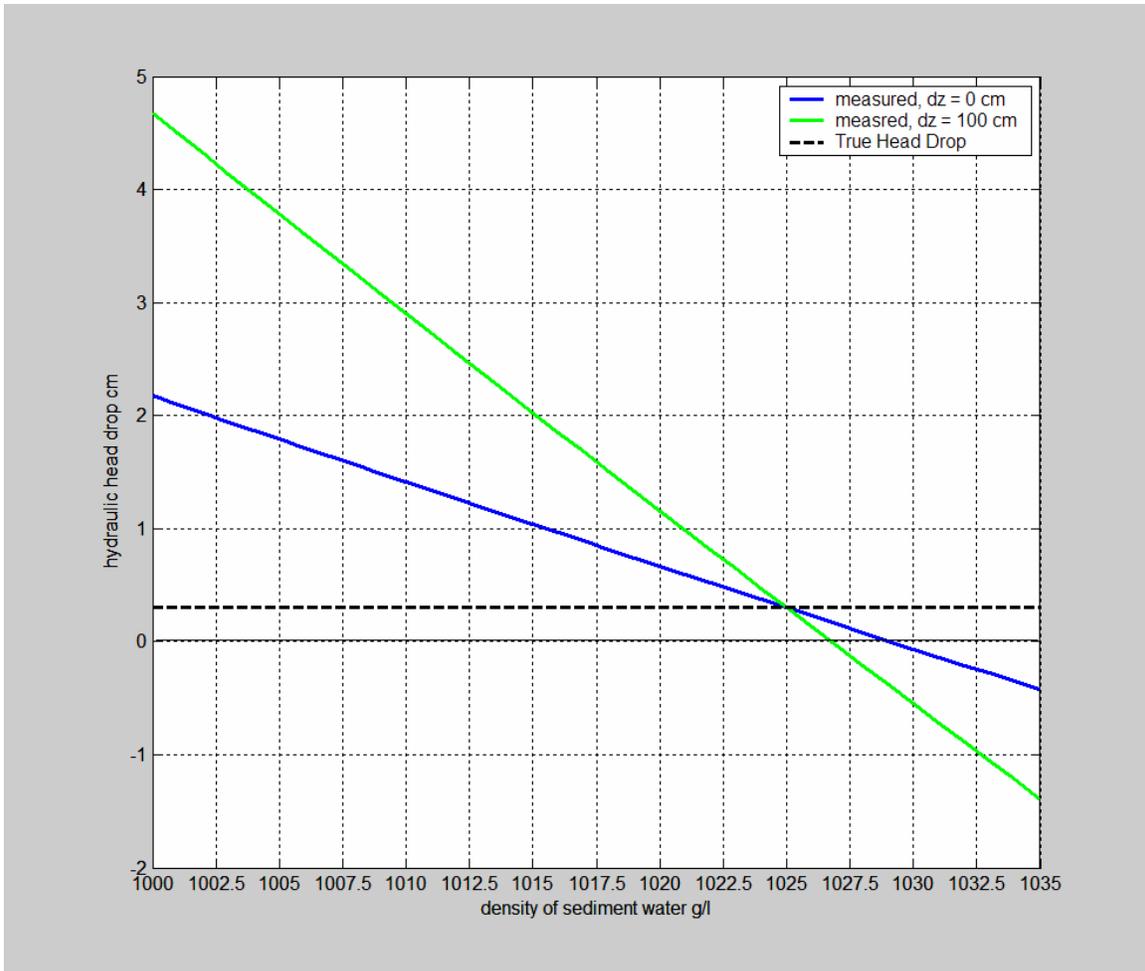
$$z_s = 0.3 + \frac{1025}{\rho_s} * 75$$

For these numbers, we can then calculate the head drop that we would measure as a function of sediment density and how high we pulled the water up and compare it to the true head drop:

$$\Delta h_{s, measured} = \left(0.3 + \frac{1025}{\rho_s} * 75 - 75 \right) + \Delta z_B \left(\frac{1025}{\rho_s} - 1 \right) \quad (8)$$

Matlab Code for Producing Graphs below:

```
dz=100;ps=[1000:1:1035];h=.3+1025./ps*75-75+dz.*(1025./ps-1);figure(1);plot(ps,h)
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To get a 1 mm measured downward head drop even though we measured an upward flux, we need the sediment water to be about 1.5 g/l denser. This density difference was within our salinity measurement error and could also result from the sediment water being about 4 degrees colder. We would need to more accurately measure salinities in the field along with the temperature of the water.

(5/5) Bonus: Waqiout Bay is very shallow and has a very small opening? Are you worried about sharks in Waqiout Bay? Is this reasonable?

You shouldn't be toooo worried, but a six footer was spotted were you took measurements