Problem Formulation:

Maximize \( F(x_1, x_2, \ldots, x_n) \)

\[ x_1, x_2, \ldots, x_n \]

such that:

- Strict equality constraints
  \[ g_i(x_1, x_2, \ldots, x_n) = 0 \quad i = 1, \ldots, r \]
- Inequality constraints
  \[ g_i(x_1, x_2, \ldots, x_n) \leq 0 \quad i = r + 1, \ldots, m \]

Basic components:

- \( n \) decision variables \( x \rightarrow x_i = [x_1, x_2, \ldots, x_n] \), collectively define a decision strategy.
- Scalar objective function \( F(x) \rightarrow F(x_1, x_2, \ldots, x_n) \) measures performance of decision strategy
- \( r \) equality constraints \( g_i(x), i = 1, \ldots, r \)
- \( n-r \) inequality constraints \( g_i(x), i = r + 1, \ldots, m \)

Note:

- Minimization of \( F(x) \) is maximization of \(-F(x)\)
- \( g(x) > 0 \) is same as \(-g(x) < 0\)

**Feasible region** \( \mathcal{F} \): Set of \( x \) that satisfies constraints (depends only on \( g_i(x) \)).

**Discrete optimization:** \( \mathcal{F} \) consists of a finite number of feasible solutions

\[
g_1(x) = -1 \text{ for } (x_1, x_2) = \{AA, AB, AC, BA, BB, CB, CC\}
g_1(x) = +1 \text{ otherwise}
\]
Continuous (non-discrete) optimization: \( \mathcal{F} \) consists of an infinite number of feasible solutions

\[ \mathcal{F} \] is bounded by curves corresponding to \( g_i(x) = 0 \).
Interior of \( \mathcal{F} \) is set of points that satisfy \( g_i(x) < 0 \).

Solving Optimization Problems

Objective in optimization is to find the best decision strategy among all feasible possibilities:

\[ \rightarrow \] We seek a global optimum

Most common way to find optimum for large problems is to use an iterative search:

An iterative search algorithm needs:
- A method for selecting an initial feasible solution - Can be formulated as a secondary optimization problem
- A stopping criterion that detects following:
  1. No feasible solution – no way to satisfy all constraints
  2. Optimal solution found – satisfies optimality conditions
  3. Objective function unbounded over feasible region - Objective can be infinite within feasible region.
• A **solution improvement mechanism** – challenging for nonlinear problems, often based on optimality conditions, sometimes *ad hoc*.

**Types of search procedures:**
- **Exhaustive Searches** - For **discrete problems**:  
  Move methodically through all (or sometimes a subset) of the feasible solutions to determine which has best objective value.
- **Selective Searches** – For **continuous problems**:  
  Use information from current and past candidate solutions (e.g. objective value or objective gradient) to determine next feasible solution.

For now, focus on continuous problems and selective searches.

**Global vs. Local Maxima for Continuous Problems**
In practice, it is much easier to find **local optima**:
- $x^*$ is a **local maximum** if $F(x^*) \geq F(x)$ for all feasible $x$ near $x^*$
- $x^*$ is a **local minimum** if $F(x^*) \leq F(x)$ for all feasible $x$ near $x^*$

Two **key questions**:
1. When is a **local** optimum also **global** optimum?
2. How do we know when a particular candidate solution $x^*$ is a **local optimum**?

What can we say about **global optimality** based on **local properties** (near $x^*$)?