6 Initial Mixing

- Introduction
- Integral Analysis
- Dimensional Analysis
- Multi-port Diffusers
- Gravitational spreading, intrusion & mixing
- Multi-port Diffusers in Shallow Water
- Buoyant Surface Jets
- Combined Near and Far Field Analysis
Submerged Discharge

- Mixing by turbulent entrainment rather than exchange
- Dilution
  - $S = Q/Q_o$
  - $S = (C_o-C_b)/(C-C_b)$
- Mixing zones
  - Hydrodynamic
  - Regulatory
Dilution a solution to pollution?

- Biodegradable contaminant?
- High ambient concentration of contaminant?
- Toxics?
Pure Jet

- Momentum driven
- Bell-shaped velocity distribution (in jet)
- Irrotational flow (entrainment field)

Properties
- $b \sim x$
- $u \sim x^{-1}$
- $Q \sim ub^2 \sim x$

Daily and Harleman, (1966)

Figure by MIT OCW.
Buoyant Jet

- Buoyancy driven
  - Temperature
  - Dissolved/Suspended solids
- Bell-shaped velocity & scalar distributions
- Linear spread
- Finite initial size (ZOFE)
Equation of State (Gill, 1982)

\[ \rho = \rho(T) + \Delta \rho(S) + \Delta \rho(TSS) \]

\[ \rho(T) = 1000 \left[ 1 - \frac{T + 288.9414}{508929.2(T + 68.12963)} (T - 3.9863)^2 \right] \]

\[ \Delta \rho(S) = AS + BS^{3/2} + CS^2 \]

\[ A = 0.824493 - 4.0899 \times 10^{-3} T + 7.6438 \times 10^{-5} T^2 - 8.2467 \times 10^{-7} T^3 + 5.3875 \times 10^{-9} T \]

\[ B = -5.72466 \times 10^{-3} + 1.0227 \times 10^{-4} T - 1.6546 \times 10^{-6} T^2 \]

\[ C = 4.8314 \times 10^{-4} \]

\[ \Delta \rho(TSS) = TSS \left[ 1 - \frac{1}{SG} \right] \times 10^{-3} \]

\[ \rho = \text{kg/m}^3, \ T \text{ in } ^\circ\text{C}, \ S \text{ in PSU (g/kg), } \text{TSS in mg/L} \]
Fischer, et al. (1979)

\[ \sigma_v = 1000(\rho - 1) \]

(\(\rho\) in g/cm\(^3\))

Figure by MIT OCW.
Model Types

- Computational Fluid Dynamics (3-D)
- Integral Analysis (1-D)
- Dimensional Analysis (0-D)
Integral Analysis: Self-Similarity

\[ \frac{\tilde{u}}{\tilde{u}_c} = f\left(\frac{r}{b}\right) \]

\[ \frac{\Delta c}{\Delta c_c} = \frac{\Delta T}{\Delta T_c} = \frac{\Delta \rho}{\Delta \rho_c} = g\left(\frac{r}{b}\right) \]

\[ f\left(\frac{r}{b}\right) = e^{-\frac{r^2}{b^2}} \]

\[ g\left(\frac{r}{b}\right) = e^{-\frac{r^2}{(\lambda b)^2}} \]
Integrated Fluxes

Volume

\[ Q \cong \int_{-\infty}^{\infty} \tilde{u} dA = \int_{0}^{\infty} \tilde{u}_c f 2\pi r dr = 2\pi I_1 \tilde{u}_c b^2 \]

Momentum* \[ M \cong \int_{-\infty}^{\infty} \tilde{u}^2 dA = \int_{0}^{\infty} \tilde{u}_c^2 f^2 2\pi r dr = 2\pi I_2 \tilde{u}_c^2 b^2 \]

Mass \[ J \cong \int_{-\infty}^{\infty} \tilde{u} \Delta c dA = \int_{0}^{\infty} \tilde{u}_c \Delta c f g 2\pi r dr = 2\pi I_3 \tilde{u}_c \Delta_c b^2 \]

Neglects turbulent momentum fluxes
Conservation Statements

Continuity
\[ \frac{dQ}{d\bar{x}} = 2\pi b |v_e| = 2\pi b \alpha \bar{u}_c \]

Longitudinal Momentum
\[ \frac{dM}{d\bar{x}} = 2\pi \int_0^\infty \Delta \rho g \sin \theta r dr = 2\pi I_4 \Delta \rho gb^2 \sin \theta \]

Horizontal Momentum
\[ \frac{d(M \cos \theta)}{d\bar{x}} = 0 \]

Contaminant mass
\[ \frac{dJ}{d\bar{x}} = 0 \]

Geometry 1
\[ \frac{dx}{d\bar{x}} = \cos \theta \]

Geometry 2
\[ \frac{dy}{d\bar{x}} = \sin \theta \]
Solution Technique

- Initial Value Problem
- 6 equations in 6 unknowns

\[
\begin{bmatrix}
\Delta \rho_c \\
\dot{u}_c \\
\dot{b} \\
\dot{\theta} \\
\dot{x} \\
\dot{y}
\end{bmatrix} = A \begin{bmatrix}
\Delta \rho_c \\
\dot{u}_c \\
\dot{b} \\
\dot{\theta} \\
\dot{x} \\
\dot{y}
\end{bmatrix} = C
\]
Results

Output as function of

\[ F_o = \frac{u_o}{\sqrt{g(\Delta \rho_o / \rho)D_o}} \]

Densimetric Froude Number

Dimensionless Distance, Height
\[ x / D_o \]
\[ z / D_o \]

Limiting Conditions
\[ F_o = \infty \quad \text{Pure jet} \]
\[ F_o = 1 \quad \text{Pure plume} \]
RNN
Temperature/ Width Chart
$\theta = 90^\circ$

Centerline Excess Temperature
$\frac{\Delta T_c}{\Delta T_o}$

Vertical Distance $Z/D$

W/D = 10
F = 600

Figure by MIT OCW.
Example Calculations (WE 6-1)

- $Q_o = 0.00125 \text{ m}^3/\text{s}$
- $D_o = 0.1 \text{ m}$
- $\Delta \rho_o/\rho = 0.025$
- $u = Q_o/(\pi D^2/4) = 0.16\text{ m/s}$
- $F_o = u_o/(\Delta \rho_o g/\rho D)^{0.5} = 1$
- $z/D_o = 70$
- $c/c_o = 0.008$
RNN
Temperature/ Width Chart
\( \theta = 90^\circ \)

Centerline Excess Temperature

\( \Delta T \frac{c}{H_0} \)

Vertical Distance \( Z/D \)

Shirazi & Davis, 1974

Figure by MIT OCW.
Example Calculations (cont’d)

<table>
<thead>
<tr>
<th></th>
<th>Base Case</th>
<th>Increased Momentum</th>
<th>Increased Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>(D_o)</td>
<td>0.1</td>
<td>0.05</td>
<td>0.1</td>
</tr>
<tr>
<td>(Q_o)</td>
<td>0.00125</td>
<td>0.00125</td>
<td>0.0025</td>
</tr>
<tr>
<td>(u_o)</td>
<td>0.16</td>
<td>0.64</td>
<td>0.032</td>
</tr>
<tr>
<td>(\Delta \rho_o/\rho)</td>
<td>0.025</td>
<td>0.025</td>
<td>0.0125</td>
</tr>
<tr>
<td>(F_o)</td>
<td>1</td>
<td>5.7</td>
<td>2.8</td>
</tr>
<tr>
<td>(z/D_o)</td>
<td>70</td>
<td>140</td>
<td>70</td>
</tr>
<tr>
<td>(c/c_o)</td>
<td>0.008</td>
<td>0.008</td>
<td>0.016</td>
</tr>
</tbody>
</table>

In deep water behavior depends mainly on buoyancy—not momentum, flow rate, port size or orientation.
Dimensional Analysis

- Identify important independent and dependent variables
- Arrange in dimensionally consistent manner
- Determine coefficients empirically
Buckingham Π Theorem

The number of dimensionless parameters equals the number of independent plus dependent variables minus the number of dimensions used to describe these variables.

Example: $D = \frac{1}{2} gt^2$

- 3 variables ($g$, $t$, $D$)
- 2 dimensions (length, time)
- 1 dimensionless variable ($D/gt^2$)

“Empirical” coefficient (1/2)
Axi-symmetric Plume

- Neglect ambient current, stratification
- Assume deep water (initial momentum, flow rate, nozzle size, discharge angle less important than buoyancy)

- Kinematic buoyancy flux
  \[ B_o = Q_o \frac{g \Delta \rho_o}{\rho} \quad [L^4 T^{-3}] \]
Axi-symmetric Plume (cont’d)

- $Q = f(B, z)$
- 3 variables – 2 dimensions = 1 non-dimensional parameter ($c_1$)

$$c_1 = \frac{Q}{B_0^\alpha z^\beta}$$

$$Q = c_1 B_0^\alpha z^\beta$$
Axi-symmetric Plume (cont’d)

\[ Q \sim B_0^\alpha z^\beta \]

\[ \frac{L^3}{T} = \frac{L^{4\alpha}}{T^{3\alpha}} L^\beta \]

\[ 3 = 4\alpha + \beta \]

\[ 1 = 3\alpha \]

\[ \therefore \alpha = \frac{1}{3}, \beta = \frac{5}{3} \]

\[ S = \frac{Q}{Q_o} \]

\[ S_c = \frac{c_1 B_o^{1/3} z^{5/3}}{Q_o} \]

\[ c_1 \approx 0.1 \]
Input variables

- \( Q_o = 0.00125 \text{ m}^3/\text{s} \)
- \( D_o = 0.1 \text{ m} \)
- \( z = 7 \text{ m} \)
- \( \Delta \rho_o/\rho = 0.025 \) (salt water-fresh water)

Derived variables

- \( B_o = Q_o \ g \ \Delta \rho_o/\rho = 0.00031 \text{ m}^4/\text{s}^3 \)
- \( F_o = u_o/(g \ \Delta \rho_o/\rho \ D_o)^{0.5} = 1 \)
- \( z/D_o = 70 \)
Integral vs Dim Anal (cont’d)

Integral Analysis
- $\Delta c_c/\Delta c_o = 0.008$
- $S_c = 125$

Dimensional Analysis
- $S_c=0.1B_o^{1/3}z^{5/3}/Q_o=138$

Figure by MIT OCW.
Blockage at surface (or trap elevation)

- Prevents entrainment of ambient water near top of trajectory
- Mixing & extra entrainment as jet “turns the corner”
- “Near field” dilution
  - $S_n = 0.26B_o^{1/3}H^{5/3}/Q_o$
  - $X_n/H = 2.8$
- $h_s/H \sim 0.11$ (horizontal discharge)
Ambient Stratification

- Stratification frequency $N$

\[ N^2 = \frac{|g \partial \rho|}{\rho \partial z} \]

- Plume traps at level of neutral buoyancy with reduced dilution

\[ h_t = 2.8 B_o^{1/4} / N^{3/4} \]

\[ S_m = 0.9 B_o^{3/4} / Q_o N^{5/4} \]
Ambient Current

- Deflects plume downstream
- Augments dilution if strong
- $S_m = 0.32 u_a H^2/Q_o$
- $x_s = 0.3 Q_o (g \Delta \rho_o / \rho) / u_a^3$
Dense plumes

Typical applications:
- Cold water from LNG terminals
- Brine from desal plants, sol’n mining of salt domes

\[
\begin{align*}
ht &= 2.3M_o^{3/4}/B_o^{1/2} \\
S_m &= 2.8M_o^{5/4}/Q_o B_o^{1/2}
\end{align*}
\]
Example: solution mining of salt domes

- **Strategic Petroleum Reserve**
  - Dates from 1970’s
  - ~700x10^6 bbl stored in 4 domes in LA & TX
  - Salinity gradients in GoM confuse shrimp

- Also used for
  - Salt production
  - Compressed gas storage
  - Waste isolation
Multi-phase Plumes

- **Bubble plumes**
  - Reservoir destratification
  - Aeration
  - Ice prevention
  - Pollutant containment

- **Droplet plumes**
  - Deep oil spills

- **Sediment plumes**
  - Dredged mat’l disposal
  - CO₂ ocean storage
Reservoir Applications

Surface Radial Jet

Thermocline

Destratification

Aeration

H ~ O(H₂)

Figure by MIT OCW.
Deep Oil-well Blowout

Figure by MIT OCW.
CO$_2$ Sequestration

Adapted from Heroz et al. (2000).
What are gas hydrates?

“Filled ice”

Example: methane hydrate

Cage structures of gas hydrates

\[
CO_2 + nH_2O \rightleftharpoons_{T,P} CO_2 \cdot nH_2O
\]

\[n \approx 5.75\]

\[\rho_h = 1100 - 1140 \text{ kg/m}^3\]
CO$_2$/seawater phase diagram

![CO$_2$/seawater phase diagram](image_url)

Figure by MIT OCW.
Laboratory studies
(Oak Ridge National Lab)

Details of mixing zone

Hydrate–liquid CO\textsubscript{2}–water composite extrusion

Liquid CO\textsubscript{2}  Water

West et al., 2003; Lee et al 2003

ORNL SPS
(Seafloor process simulator)
Two-phase plume model

(100 kg/s CO₂, 1 cm diameter spheres, release depth 800 m, \( \frac{Q_c}{Q_w} = \lambda = 0.49 \))
Multi-port diffusers

**Construction:**
- Cut and cover
- Bored tunnel

**Ports**
- \( l \sim 0.3H \) (or 0.3 h)
- Often 2 or more per riser

**Line source approx.**
- \( q_o = Q_o/L, \; b_o = B_o/L \)

**No current; no strat**
- \( S_m = 0.42Hb_o^{1/3}/q_o \)

**No current; strat**
- \( S_m = 0.97b_o^{2/3}/q_o N \)

**Current; strat**
- \( S_m = 2.2u_a^{1/2}b_o^{1/2}/Nq_o \)
Single vs Multiport (WE 6-3)

Boston Outfall

- Diffuser Length $L = 2000$ m
- No ports $N_p = 440$
- Flow rate $Q_o = 20$ m$^3$/s
- Water depth $H = 30$ m
- Stratification frequency $N^2 = \left| \frac{g \partial \rho}{\rho \partial z} \right|

- $N^2 = \frac{(9.8)(25-22)}{(1025)(30)} = 0.001$ s$^{-2}$
Figure 1
CROSS-SECTION OF SEAWATER DENSITY ALONG NORTHERN TRANSECT (Units of Sigma-t): 8/12/87 AM
As single port

- $Q_o = \frac{20}{440} = 0.045 \text{ m}^3/\text{s}$
- $B_o = 0.045 \times 9.8 \times 0.025 = 0.011 \text{ m}^4/\text{s}^3$
- $h_t = 2.8B_o^{1/4}/N^{3/4} = 12 \text{ m}$
- $\ell = L/N_p = \frac{2000}{440} = 4.5 \text{ m}$
  - $\ell > 0.3 h_t \Rightarrow \text{no merging}$
- $S_m = 0.9 B_o^{3/4}/Q_o N^{5/4} =$
  
  $0.9(0.011)^{3/4}/(0.045)(0.0013)^{5/8} = 51$
As multi-port diffuser (line source of buoyancy)

- \( q_0 = \frac{20}{2000} = 0.01 \text{ m}^2/\text{s} \)
- \( b_0 = 0.01 \times 0.025 \times 9.8 = 0.0025 \text{ m}^3/\text{s}^3 \)
- \( h_t = 2b_0^{1/3}/N = 2(0.0025)^{1/3}/(0.001)^{1/2} = 9 \text{ m} \)
- \( S_m = 0.97b_0^{2/3}/q_0N = 0.97(0.0025)^{2/3}/(0.01)(0.001)^{1/2} = 56 \)
Numerical modeling of sewage outfalls?

MWRA, 1999
Numerical modeling of sewage outfalls?

MWRA, 1999
Gravitational spreading, intrusion, mixing

**Surface spreading layer**

\[ h_s = \frac{u_a^2}{g_N'} \]

\[ x_s = 0.3Q_N g_N' / u_a^3 \]

\[ b_s = 0.8Q_N g_N' / u_a^3 \]

**Internal spreading layer**

\[ h_s = 1.2u_a / N \]

\[ x_s = 0.25Q_N N / u_a^2 \]

\[ b_s = 0.65Q_N N / u_a^2 \]
Neutrally buoyant jet in stratification

\[
x_m = 3.5 M_o^{1/4} / N^{1/2}
\]
\[
S_m = 0.63 M_o^{3/4} / N^{1/2} Q_o
\]
\[
h_m = 0.95 M_o^{1/4} / N^{1/2}
\]
Occasional taste and odor problems
- Synura (left)
- Chrysosophaerella

Algal locations
- Hypolimnion
- Metalimnion
- Under ice

Conventional treatment (surface algae) with CuSO₄ from boat

How to efficiently treat (place algaeicide in proper stratum) under ice & at depth?
Layout of Treatment System
(potential system being discussed)

Figure by MIT OCW.
Mid-Depth Air Driven Circulator

Chemical Injection

Air Diffusers

3 meters

10 meters

CDM, 2005
Application at Depth

Length, thickness and dilution (hence required operation time) depend on reservoir stratification and discharge momentum.
Application under Ice

Relies on bubble plume to transport algaecide to surface

Figure 5
Multi-port diffusers in shallow water

- **Typical for power plant (thermal) discharges**

\[
S_a = \sqrt{\frac{0.26g^{2/3}H^2L^{4/3}}{Q_o^{4/3}}} + \left(\frac{u_a HL}{Q_o}\right)^2
\]

\[
S_s = \frac{0.5u_a HL}{Q_o} + \sqrt{\left(\frac{0.5u_a HL}{Q_o}\right)^2 + \frac{0.19HLu_o}{Q_o}}
\]

\[
S_t = \sqrt{\frac{HLu_a^2}{2Q_o u_o + 10u_a^2 HL}}
\]

\[
S_c = \frac{0.5u_a HL}{Q_o} + \sqrt{\left(\frac{0.5u_a HL}{Q_o}\right)^2 + \frac{0.5HLu_o}{Q_o}}
\]

Figure by MIT OCW.
Buoyant surface discharges

- Thermal plumes and river discharges
- Independent variables
  \[ F'_o = \frac{u_o}{\sqrt{g(\Delta \rho_o / \rho)\ell_o}} \]
  \[ \ell_o = \sqrt{h_o b_o} \]
- Dependent variables
  - \( S = 1.4F'_o \)
  - Lengths \( \sim F'_o \ell_o \)

Figure by MIT OCW.
Combined near and far field analysis (accounting for background build-up)

Far Field Dilution

\[ S_F = \frac{c_o - c_a}{c_F - c_a} \]

Near Field Dilution

\[ S_N = \frac{c_o - c_F}{c_N - c_F} \]

Total Dilution

\[ S_T = \frac{c_o - c_a}{c_N - c_a} \]

\[ \frac{1}{S_T} = \frac{1}{S_N} + \frac{1}{S_F} - \frac{1}{S_N S_F} \approx \frac{1}{S_N} + \frac{1}{S_F} \]

Total dilution less than either near field or far field dilution and controlled by the smaller of the two.
Example

- Far field dilution $S_F = 50$ to $100$
- Near Field dilution $S_N = 50$ to $100$
- Total Dilution $S_T = 25$ to $33$ to $50$

MWRA, 1999