Problem set 4: Convection and moist processes  
Due date: October 8th, 2008

1. In class, we showed that the pressure of an isothermal atmosphere varies exponentially with height. Consider now an atmosphere with uniform potential temperature. Find how pressure varies with height, and show in particular that such an atmosphere has a discrete top (where \( p \to 0 \)) at altitude \( RT_0/(\kappa g) \), where \( R \), \( \kappa \), and \( g \) have their usual meanings, and \( T_0 \) is the temperature at 1000hPa pressure.

2. Consider an isothermal atmosphere in hydrostatic balance, with temperature \( T_0 = 280 \text{K} \). Suppose an air parcel is moved adiabatically from the surface to an altitude of 10km. Determine its temperature on arrival
   (a) assuming its potential temperature is conserved during adiabatic displacement;
   (b) assuming its temperature decreases with altitude at the adiabatic lapse rate
   \[
   \frac{\partial T}{\partial z} = -\frac{g}{c_p}.
   \]
   Compare and discuss. Now repeat the calculations for an atmosphere which has the same surface temperature, but in which the potential temperature is constant with height.

3. Use the expression for saturated specific humidity:
   \[
   q_s = \left(\frac{R}{R_v}\right) \frac{e_s(T)}{p}
   \]  
   (1)
   and the following empirical (but pretty accurate) relation for saturated vapor pressure \( e_s(T) \):
   \[
   e_s = A \exp(\beta T)
   \]  
   (2)
   where \( A = 6.11 \text{ hPa} \) and \( \beta = 0.067 \text{C}^{-1} \) and \( T \) is in °C, to compute from tabulated data of \( T(p) \) (use annual means), vertical profiles of saturated specific humidity, \( q_s(p) \). Carry out your calculation using \( T(p) \) data at the equator and at 50°N. You will need to look up values of \( R \) and \( R_v \) in your notes.
   Compare your \( q_s \) profiles with observed profiles of \( q \), the specific humidity, at the same latitudes. Comment?
   You can obtain tabulated data here (follow detailed instructions in appendix):
   http://paoc.mit.edu/labweb/atmos-obs/temperature.htm and
   http://paoc.mit.edu/labweb/atmos-obs/specifichumidity.htm
Appendix

To obtain data from the web, follow this example. Suppose you want to obtain a table of annual mean $T(p)$ at 50°N. Go to http://paoc.mit.edu/labweb/atmos-obs/observations.htm and click on temperature on the side bar. Click on the annual mean T picture and you will taken through to Ingrid, a data server at Lamont. Underneath the figure, in the Get Data row, click Entire data set, then click tables (toward the top on the right) and then P,Y Table. The tabulated data should pop up in your browser. Pick out the 50°N (not south!!) row. That’s it. The procedure is the same for other data. E-mail professor if you have problems.