Seismic waves
Seismic wave equation

\[
\text{mass } \times \text{acceleration} = \sum \text{forces}
\]

\[
\rho \frac{\partial^2 u_i}{\partial t^2} = \sigma_{ij,j} + f_i
\]

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Stein and Wysession, 2003
Seismic wave equation

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Stein and Wyssession, 2003

strain tensor

\[ e_{ij} = \frac{1}{2} \left( \partial_i u_j + \partial_j u_i \right) = \begin{pmatrix} \frac{\partial u_1}{\partial x_1} & \frac{1}{2} \left( \frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1} \right) & \frac{1}{2} \left( \frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1} \right) \\ \frac{1}{2} \left( \frac{\partial u_2}{\partial x_1} + \frac{\partial u_1}{\partial x_2} \right) & \frac{\partial u_2}{\partial x_2} & \frac{1}{2} \left( \frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2} \right) \\ \frac{1}{2} \left( \frac{\partial u_3}{\partial x_1} + \frac{\partial u_1}{\partial x_3} \right) & \frac{1}{2} \left( \frac{\partial u_3}{\partial x_2} + \frac{\partial u_2}{\partial x_3} \right) & \frac{\partial u_3}{\partial x_3} \end{pmatrix} \]
Seismic wave equation

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Stein and Wysession, 2003

strain tensor

\[ e_{ij} = \frac{1}{2} ( \partial_i u_j + \partial_j u_i ) = \begin{pmatrix}
\frac{\partial u_1}{\partial x_1} & \frac{1}{2} \left( \frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1} \right) & \frac{1}{2} \left( \frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1} \right) \\
\frac{1}{2} \left( \frac{\partial u_2}{\partial x_1} + \frac{\partial u_1}{\partial x_2} \right) & \frac{\partial u_2}{\partial x_2} & \frac{1}{2} \left( \frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2} \right) \\
\frac{1}{2} \left( \frac{\partial u_3}{\partial x_1} + \frac{\partial u_1}{\partial x_3} \right) & \frac{1}{2} \left( \frac{\partial u_3}{\partial x_2} + \frac{\partial u_2}{\partial x_3} \right) & \frac{\partial u_3}{\partial x_3}
\end{pmatrix} \]
Seismic wave equation

Constitutive equation (Hooke’s Law)

\[
\sigma_{ij} = c_{ijkl} e_{kl} = \sum_{k=1,3} \sum_{l=1,3} c_{ijkl} e_{kl}
\]

\[
\sigma_{ij} = c_{ijkl} \sum_{k=1,3} \sum_{l=1,3} \left[ \frac{1}{2} \left( \partial_k u_l + \partial_l u_k \right) \right]
\]

\[
= c_{ijkl} \sum_{k=1,3} \sum_{l=1,3} [\partial_l u_k]
\]

\[
= c_{ijkl} u_{k,l}
\]

Back to the equation of motion:

\[
\text{mass} \times \text{acceleration} = \sum \text{forces}
\]

\[
\rho \frac{\partial^2 u_i}{\partial t^2} = \sigma_{ij,j} + f_i
\]

\[
\rho \frac{\partial^2 u_i}{\partial t^2} = \left[ c_{ijkl} u_{(k,l)} \right]_j + f_i
\]

\[
\rho \frac{\partial^2 u_i}{\partial t^2} \approx c_{ijkl} u_{(k,l),j} + f_i
\]
Seismic wave equation

\[ \rho \frac{\partial^2 u_i}{\partial t^2} = c_{ijkl} u_{(k,l),j} + f_i \]

isotropic medium:

\[ c_{ijkl} = \lambda \delta_{ij} \delta_{kl} + \mu (\delta_{ij} \delta_{jk} + \delta_{ik} \delta_{jl}) \]

Helmholtz decomposition:

\[ u = \nabla \phi + \nabla \times \psi \]

\[ \nabla \times \nabla \phi = 0 \]

\[ \nabla \cdot \nabla \times \psi = 0 \]
Seismic wave equation

P-wave

wave eq: \[ \alpha^2 \nabla^2 \phi - \frac{\partial^2 \phi}{\partial t^2} = -\frac{1}{\rho} F_P \]

velocity: \[ \alpha = \sqrt{\frac{\lambda + 2\mu}{\rho}} \]

S-wave

\[ \beta^2 \nabla^2 \psi - \frac{\partial^2 \psi}{\partial t^2} = -\frac{1}{\rho} F_S \]

\[ \beta = \sqrt{\frac{\mu}{\rho}} \]

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Stein and Wysession, 2003
Seismic velocities in the Earth

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Seismic velocities in the Earth

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Seismic waves

Body Waves: P-waves and S-waves are body waves, as they can travel in all directions through an elastic volume.
Seismic waves

Ray theory: seismic wavefield can be described by discrete, linear ray paths linking sources and receivers (infinite frequency approximation)

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Surface waves

Surface

$\beta_1 \otimes \text{SH} \beta_2$

Velocity: $c_x < \beta_2$
Surface waves

Velocity: $c_x = 0.92 \beta$
Energy of seismic waves

\[ \overline{E} = \frac{1}{2} \rho A^2 \omega^2 \]

Body waves

\[ \overline{E}(r) \propto \frac{1}{r^2} \]

Surface waves

\[ \overline{E}(r) \propto \frac{1}{r} \]
Kobe earthquake, Jan 17, 1995, M 7.2

Image courtesy of Oklahoma Geological Survey.
Kobe earthquake, Jan 17, 1995, M 7.2

links to video footage of surface waves:

http://www.youtube.com/watch?v=pXATR6vOcfQ

http://www.youtube.com/watch?v=0plbf5w0sbA&NR=1
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Spring 2010

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