1. Geothermometry and Geobarometry – an example using mineral solutions. This exercise illustrates how you can use mineral compositions in metamorphic rocks to obtain estimates of the P-T conditions under which rocks form in the earth’s interior. This example is discussed in Spear’s book on p. 515-536 and is a metamorphic rock from New England that contains the phases: biotite, plagioclase, garnet, quartz, and the mineral kyanite. In addition, rocks in the immediate vicinity of this sample contain andalusite and sillimanite. So, this implies that the conditions of formation of the sample are very near the pressure and temperature of the invariant equilibrium in the Al$_2$SiO$_5$ one-component system (the triple point).

The accompanying Table from Spear contains the compositions of the minerals that are found in this rock. Two equilibria are used to estimate the conditions. They are:

\[
\text{Anorthite} = \text{Grossular} + \text{Kyanite} + \text{Quartz} \quad (1)
\]

\[
3 \text{CaAl}_2\text{Si}_2\text{O}_8 = \text{Ca}_3\text{Al}_2\text{Si}_3\text{O}_{12} + 2 \text{Al}_2\text{SiO}_5 + \text{SiO}_2
\]

This reaction is called the GASP reaction and the expression of the equilibrium constant is:

\[
0 = -48357 + 150.66 T \text{(^°K)} + (P-1) \text{(bars)} (-6.608) + RT \ln K_{eq}
\]

and

\[
\text{Fe}_3\text{Al}_2\text{Si}_3\text{O}_{12} + \text{KMe}_3\text{AlSi}_3\text{O}_{10}(\text{OH})_2 = \text{Mg}_3\text{Al}_2\text{Si}_3\text{O}_{12} + \text{KFe}_3\text{AlSi}_3\text{O}_{10}(\text{OH})_2
\]

\[
\text{Almandine} + \text{Phlogopite} = \text{Pyrope} + \text{Annite} \quad (2)
\]

This is an exchange reaction and the symbol used for the equilibrium constant is $K_D$. The acronym for this reaction is GARB.

The expression for this equilibrium is:

\[
52,112 - 19.51 T \text{(^°K)} + 0.238 P \text{(bars)} + 3 RT \ln K_D = 0
\]

Apply these two thermometers and determine the conditions of P and T indicated by the minerals. Assume that the mineral solutions are ideal.

For example, that means that for the GASP reaction, the activity of grossular in garnet is:

\[
a_{\text{Gross}}^{\text{Garnet}} = (X_{\text{Gross}}^{\text{Garnet}})^3
\]

\[
a_{\text{An}}^{\text{Plag}} = X_{\text{An}}^{\text{Plag}}
\]
For the GARB reaction the activities of the components are given by their Fe/(Fe+Mg) and Mg/(Mg+Fe) values:

\[ a_{\text{Alm}}^{\text{Garnet}} = X_{\text{Fe}}^{\text{Garnet}} = \frac{\text{Fe}}{\text{Fe+Mg}} \]

and

\[ a_{\text{Pyr}}^{\text{Garnet}} = X_{\text{Mg}}^{\text{Garnet}} = (1 - \frac{\text{Fe}}{\text{Fe+Mg}}). \]

So, work through this example and calculate P and T using both expressions. How close can you get to the aluminosilicate triple point?