**Problem Set 1 (Due on Wednesday, Feb. 28, 2007)**

All the figures in this PS are taken from *Introduction to Seismology* [Peter M. Shearer, 1999].

1. Let \( U = 2 \frac{x^2}{y} + 2 \frac{xy^3}{z^2} + 3xz^4 \) and \( \vec{B} = \frac{y^3}{x^2} \hat{x} + 2y^3z\hat{y} + 2 \frac{y^3z^3}{x^2} \hat{z} \)

   (a) Calculate the gradient of \( U \).

   (b) Calculate the divergence of \( \vec{B} \).

   (c) Calculate the Laplacian of \( U \).

   (d) Calculate the curl of the gradient of \( U \).

   (e) Calculate \( \vec{B} \cdot (\vec{B} \times \nabla U) \).

2. (a) Using \( c_{ijkl} = \lambda \delta_{ij} \delta_{kl} + \mu (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}) \), \( \Delta = u_{kk} \), \( \varepsilon_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}) \), and \( \tau_{ij} = c_{ijkl} \varepsilon_{kl} \), show that \( \tau_{ij} = \lambda \delta_{ji} \Delta + 2 \mu \varepsilon_{ij} \).

   (b) If \( \ddot{\mathbf{u}} = (\lambda + \mu) \nabla (\nabla \cdot \mathbf{u}) + \mu \nabla^2 \mathbf{u} \), decompose into a wave equation for a propagating volume change \( \nabla \cdot \mathbf{u} \) and a rotation \( \nabla \times \mathbf{u} \), by using the following vector identities:

\[
\nabla^2 \mathbf{u} = \nabla (\nabla \cdot \mathbf{u}) - \nabla \times \nabla \times \mathbf{u}, \quad \nabla \cdot \nabla \times \mathbf{a} = 0 \quad \text{and} \quad \nabla \times \nabla \phi = 0,
\]

where \( \mathbf{a} \) is a general vector field and \( \phi \) a general scalar field.

3. The radii of the Earth, Moon, and Sun are 6,371 km, 1,738 km, and 695100 km, respectively. From Figures 1.1, 1.5 and 1.6, make a rough estimate of how long it takes a P-wave to traverse the diameter of each body.

4. Assume that the S velocity perturbation plotted at 200km depth in Figure 1.7 extend throughout the uppermost 300 km of the mantle. Estimate how many seconds earlier a vertically upgoing S-wave will arrive at a seismic station in the middle of Canada, compared to a station in the eastern Pacific. Ignore any topographic or crustal thickness differences between the sites; consider only the integrated travel time difference though the upper mantle.

5. Assuming that the P velocity in the ocean is 1.5 km/s, estimate the minimum and maximum water depths shown in Figure 1.8. If the crustal P velocity is 5 km/s, what is the depth to the top of the magma chamber from the sea floor?