12.510 Introduction to Seismology
Spring 2008

This is the last lecture in 12.510 2008. It includes a review of the moment tensor, focal mechanisms, radiation patter, seismic source, magnitude, magnitude saturation, and moment magnitude.

A quick review of the Moment Tensor and focal mechanisms

For a point source at $(\grave{\boldsymbol{x}}, \grave{\boldsymbol{t}})$ the solution for the equation of motion is expressed with the Green's function $\boldsymbol{G}(\boldsymbol{x}, \boldsymbol{t})$*:*

$$\underbrace{u(x,t)}_{Displacemet} = \int_{intgrate\ \ over\ the\ spacial\ \ time\ \ function} \underbrace{d\grave{t}} \iiint_{spatial\ \ extent\ \ of\ source} G(x, t-\grave{t}, x, 0) * \underbrace{f(\grave{x}, \grave{t})}_{source\ \ function} \ d\grave{x}. \quad (1)$$

The force is described by the following relation:

$$f_i = A\delta(x - \grave{x}) * \delta(t - \grave{t}) * \delta_{in}\ ; \quad (2)$$

where$A$ is the amplitude, $(\boldsymbol{t}, \grave{\boldsymbol{t}})$is the time, $(\boldsymbol{x}, \grave{\boldsymbol{x}})$ is the position, and n is the direction. Substituting equation (2) in the equation of motion and then solving for the displacement (u) resulting from the wave motion due to a point source, leads to the following relations:

$$\begin{gathered} u_i(x,t) = G_{ij}(x, t; \grave{x}, \grave{t}) f_j(\grave{x}, \grave{t}), \\ u\ \ (x,t) \sim G(x, t; \grave{x}, \grave{t});\ u(x,t) = \frac{\partial}{\partial x_h} * G_{ij}(x, t, \grave{x}, \grave{t}) M_{jk}\ ; \end{gathered} \quad (3)$$

where u_i is the displacement, f_j is the force vector. Green's function gives the displacement at point x that results from the unit force function applied at point ẋ . Internal forces f must act in opposing directions $-f$, at a distance d so as to conserve momentum (force couple). For angular momentum conservation, a complementary couple balances the double couple forces. Figure 1 shows nine different force couples for the components of the moment tensor.

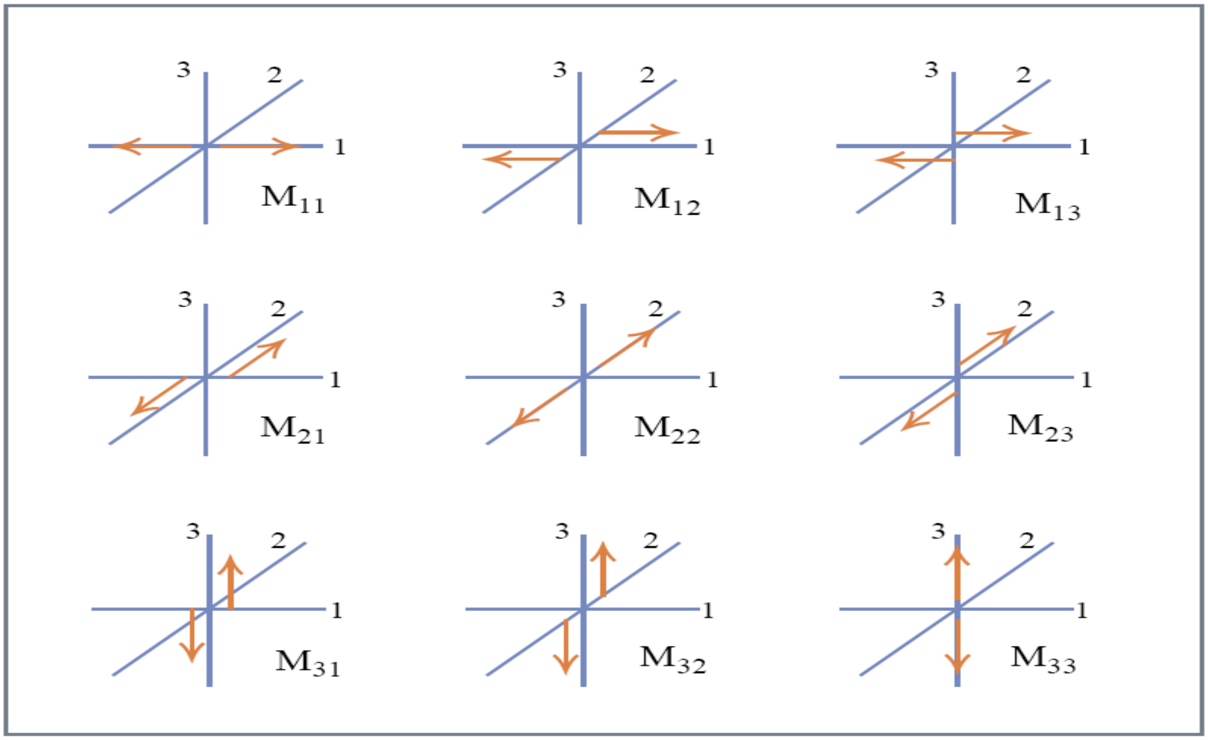


Figure by MIT OCW.

Figure 1: Different force couples for the components of the moment tensor (Source: Shearer, 1999)

Single couple **Double force couple**

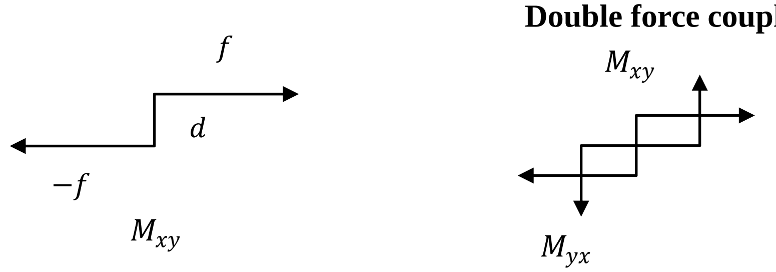


Figure 2: Single and double couple

Conserve momentum (angular) ➔ force couple

Moment Tesnor

Starting with defining the moment tensor as:

$$M_{ij} = \begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{pmatrix}; \tag{4}$$

where M_{ij} represents a pair of opposing forces pointing in the direction, separated in the j direction. Its magnitude is the product fd [unit: Nm] which is called seismic moment.

For angular momentum conservation, the condition $M_{ij} = M_{ji}$ should be satisfied, so the momentum tensor is symmetric. Therefore we have only six independent elements. This moment tensor represents the internally generated forces that can act at a point in an elastic medium. The displacement for a force couple with a distance d in the x_k direction is given by

$$u_i(x,t) = G_{ij}(x,t,\grave{x},\grave{t})f_j(\grave{x},\grave{t}) - G_{ij}(x,t,x-\grave{x}d,\grave{t})f_j(\grave{x},\grave{t}) = \frac{\partial G_{ij}(x,t,\grave{x},\grave{t})}{\partial x_k} f_j(\grave{x},\grave{t})d. \tag{5}$$

The last term can be replaced by moment tensor to get the displacement u,

$$u_i(x,t) = \frac{\partial G_{ij}(x,t,\grave{x},\grave{t})}{\partial x_k} M_{jk}(\grave{x},\grave{t}). \tag{6}$$

There is a linear relationship between the displacement and the components of the moment tensor that involves the spatial derivatives of the Green's functions. We can see the internal force f is proportional to the spatial derivative of moment tensor when compared equation (3a) with (6).

Seismic moment

$$f_j \sim \frac{\partial}{\partial x_k} M_{jk} \tag{7}$$

Let's consider a right-lateral movement on a vertical fault oriented in the x_1 direction and the corresponding moment tensor is given by

$$M = \begin{pmatrix} 0 & M_{12} & 0 \\ M_{21} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & M_0 & 0 \\ M_0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{8}$$

where $M_0 = \mu D_s$ called scalar seismic moment which is the best measure of earthquake size and energy release, μ is shear modulus, $D = D_s/L$ is average displacement, and s is area of the fault. M_0 can be time dependent, so $M_0 = \mu D(t)s(t)$. The right-hand side time dependent terms become source time function, $x(t)$, thus the seismic moment function is given by:

$$M(t) \sim \mu D(t)s(t). \tag{9}$$

We can diagonalize the moment matrix (equation 8) to find principal axes. In this case, the principal axes are at 45° to the original x_1 and x_2 axes, we get:

$$\grave{M} = \begin{pmatrix} M_0 & 0 & 0 \\ 0 & M_0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{10}$$

Principal axes become tension and pressure axis. The above matrix represents that $1x'$ coordinate is the tension axis, T, and $2x'$ is the pressure axis, P. (see Figure 3)

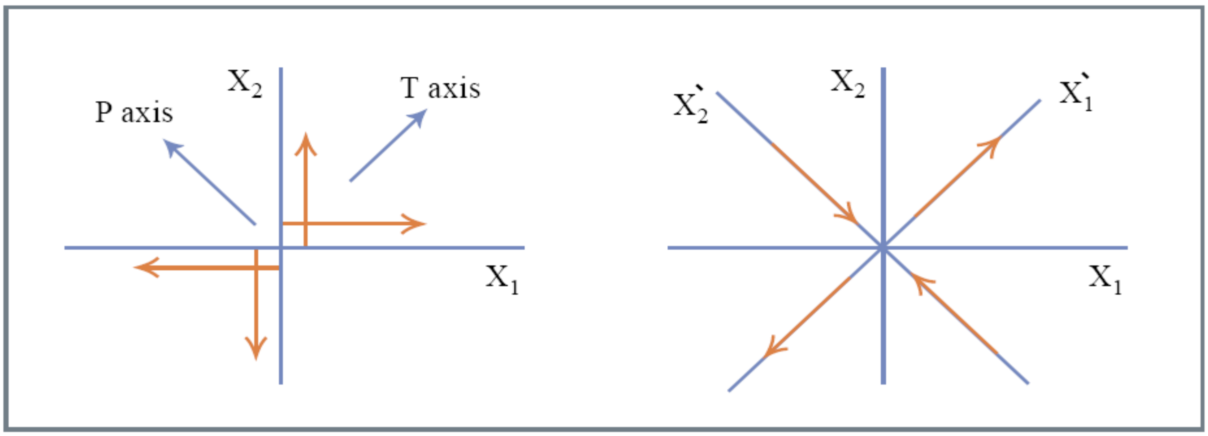


Figure by MIT OCW.

Figure 3: The double-coupled forces and their rotation along the principal axes.(Source: Shearer, 1999)

Radiation Patterns

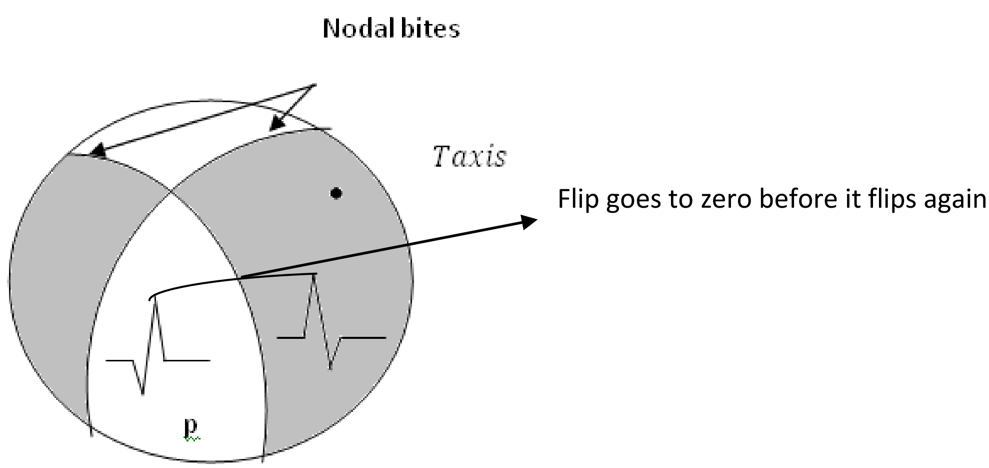
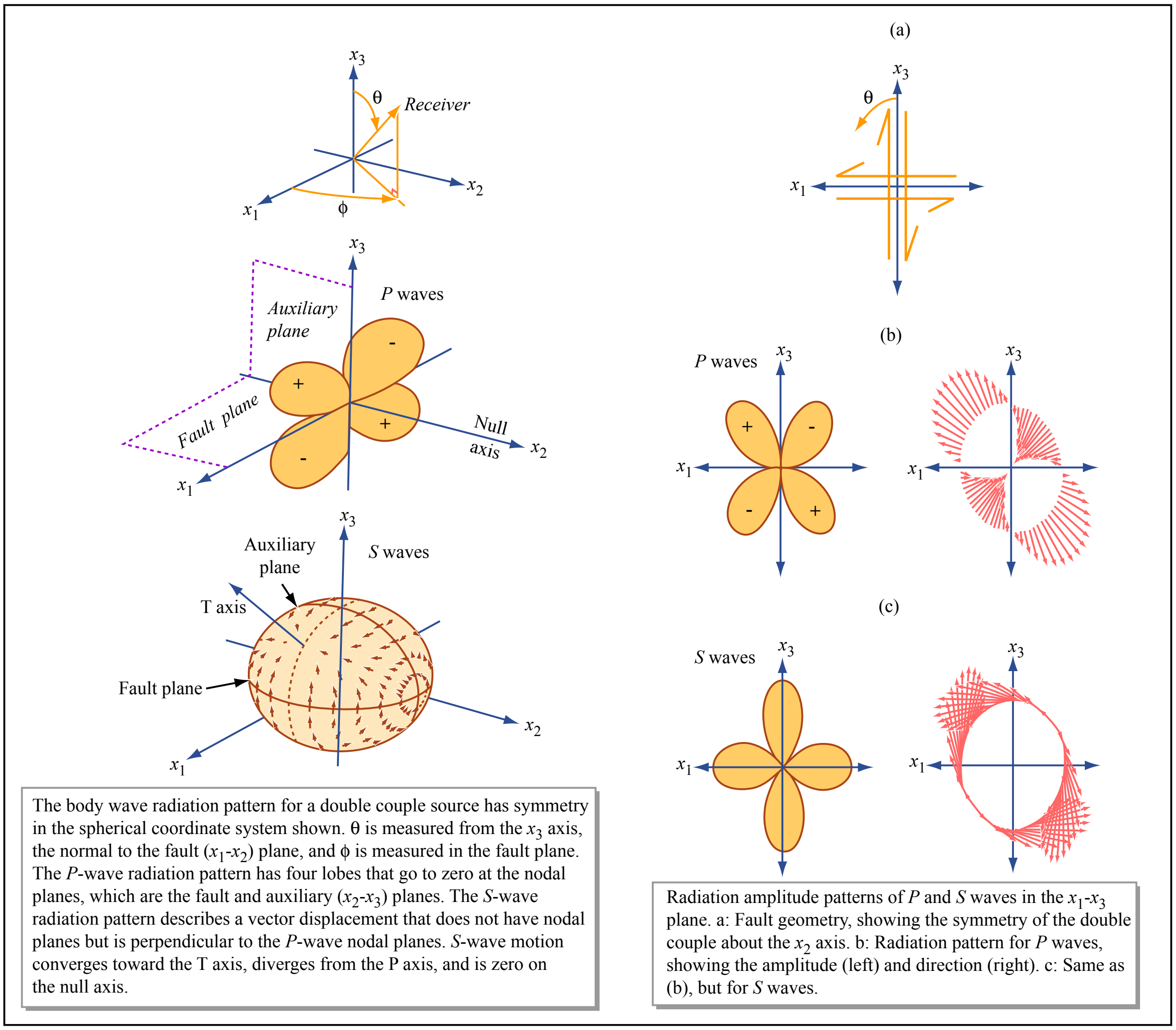


Figure 4: Radial componet

The following figure shows the variation of the radiation patterns with the direction of the receiver. It assumed that the radiation field is in spherical coordinates (will be explained in this section), where θ is measured from the z axis and $\emptyset$ is measured in the x-y plane.



The body wave radiation pattern for a double couple source has symmetry in the spherical coordinate system shown. θ is measured from the x_3 axis, the normal to the fault (x_1-x_2) plane, and ϕ is measured in the fault plane. The *P*-wave radiation pattern has four lobes that go to zero at the nodal planes, which are the fault and auxiliary (x_2-x_3) planes. The *S*-wave radiation pattern describes a vector displacement that does not have nodal planes but is perpendicular to the *P*-wave nodal planes. *S*-wave motion converges toward the T axis, diverges from the P axis, and is zero on the null axis.

Radiation amplitude patterns of *P* and *S* waves in the x_1-x_3 plane. a: Fault geometry, showing the symmetry of the double couple about the x_2 axis. b: Radiation pattern for *P* waves, showing the amplitude (left) and direction (right). c: Same as (b), but for *S* waves.

Figure by MIT OpenCourseWare.

Figure 4: The left graph shows body wave radiation patter for a double couple source. The right graph shows the radiation amplitude patterns of P and S waves in the x-z plane. (Source: Stein and Wysession, p221)

P-wave potential in spherical coordinate is given by d'Alembert

$$\phi(x,t) = f\left(t - \frac{x}{\alpha}\right) + g\left(t + \frac{x}{\alpha}\right);$$
$$\alpha = \text{P-wave speed},$$
$$\ddot{\phi} = \alpha^2 \nabla^2 \emptyset\,; \qquad (11)$$

r is the distance from the point source, and τ is time residual. Therefore, the displacement field is given by the gradient of the displacement potential $u = \frac{\partial \phi}{\partial r}$,

Spherical medium

In spherical coordinates, far field displacement is given by

$$\emptyset(x,t) = \frac{f(t - \frac{r}{\alpha})}{r}; \tag{12}$$

where r is the distance from the point source, and τ is time residual.

Then we use Helmholtz potential;

$$u = \nabla\emptyset + \nabla \times \psi, \nabla.\psi = 0, \tag{13}$$

the displacement field is given by the gradient of the displacement potential $u = \frac{\partial \phi}{\partial r}$, which leads to the following equation:

$$u(x,t) = \underbrace{\left(\frac{1}{r^2}\right) f(\tau)}_{I} + \underbrace{\left(\frac{1}{r_\alpha}\right) \frac{\partial f(\tau)}{\partial \tau}}_{II}. \tag{14}$$

The first term in the right hand side is near field displacement because of the decay as $\frac{1}{r^2}$ and the second term is far field displacement with the decay as $\frac{1}{r}$. When we consider the relation between internal force and moment tensor given by equation (9), we can find that the near field term has no time dependence but the far field term has time dependence. The relations are given by:

$$f \sim \frac{\partial M}{\partial x}, \frac{\partial f}{\partial \tau} \sim \frac{\partial M}{\partial \tau} \sim M(t) \tag{15}$$

Therefore, the near field term represents the permanent static displacement due to the source and the far field term represents the dynamic response or transient seismic waves that are radiated by the source that cause no permanent displacement. Figure 5 represents the near and far field behaviors.

in summary

I $\sim \frac{1}{r}$ near field solution ➔Static displacement
II $\sim \frac{1}{r}$ far field solution ➔ transit displacement
$\mathrm{M(t)}$ = moment time function

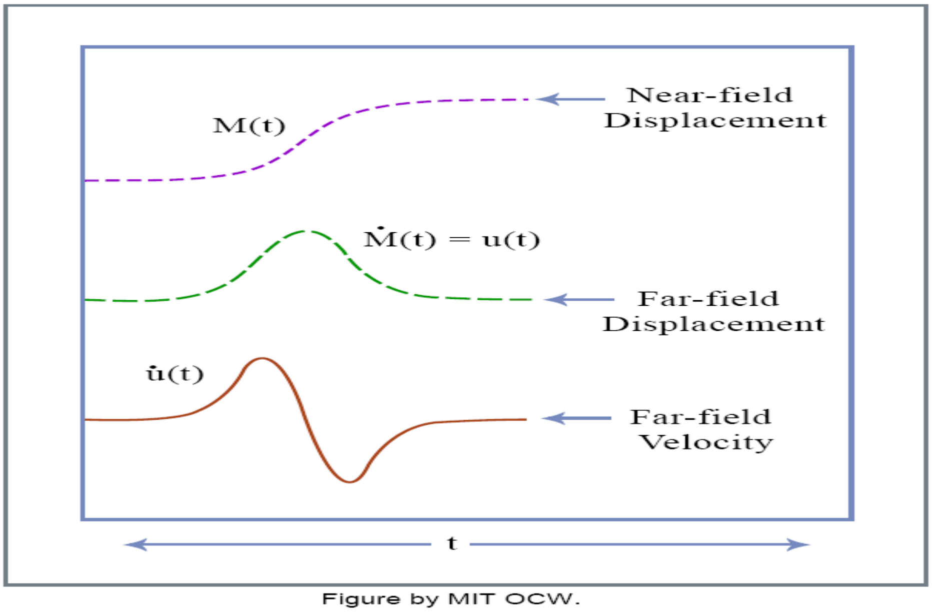


Figure 5: The relationships between near-field and far-field displacement and velocity (Source: Shearer, 1999).

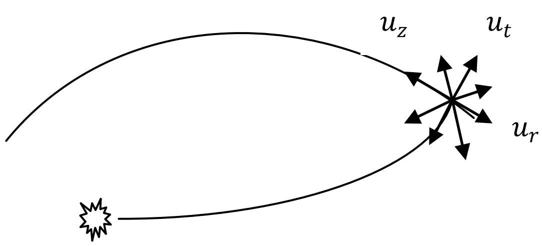


Figure 6: Radial component

In spherical coordinates, the far field displacement is given by:

$$u_r(x,t) = \frac{1}{4\pi\rho\alpha^2 r}\dot{M}(t-\frac{r}{\alpha})\underbrace{\sin 2\theta \cos\varphi}_{radiation\ \ pattern}, \tag{16}$$
$$\sim \frac{1}{r}\dot{M}(t-\frac{r}{\alpha}).$$

The first amplitude term decays as $\frac{1}{r}$. The second term reflects the pulse radiated from the fault, $\dot{M}(t)$, which propagates away with the P-wave speed α and arrives at a distance r at time $t-\frac{r}{\alpha}$. $\dot{M}(t)$ is called the seismic moment rate function or source time function. Its integration form in terms of time is given by equation (9). The final term describes the P-wave radiation pattern depedig on the two nodal planes. The first term describes P-wave radiation pattern depending on the two angles (θ, ϕ). At $\theta = \phi = 90^{\circ}$, the siplacement is zero on the two nodal planes. The maximum amplitudes are between the two nodal planes. Figure 8 shows the far-field radiation pattern for P-waves and S-waves for a double-couple source.

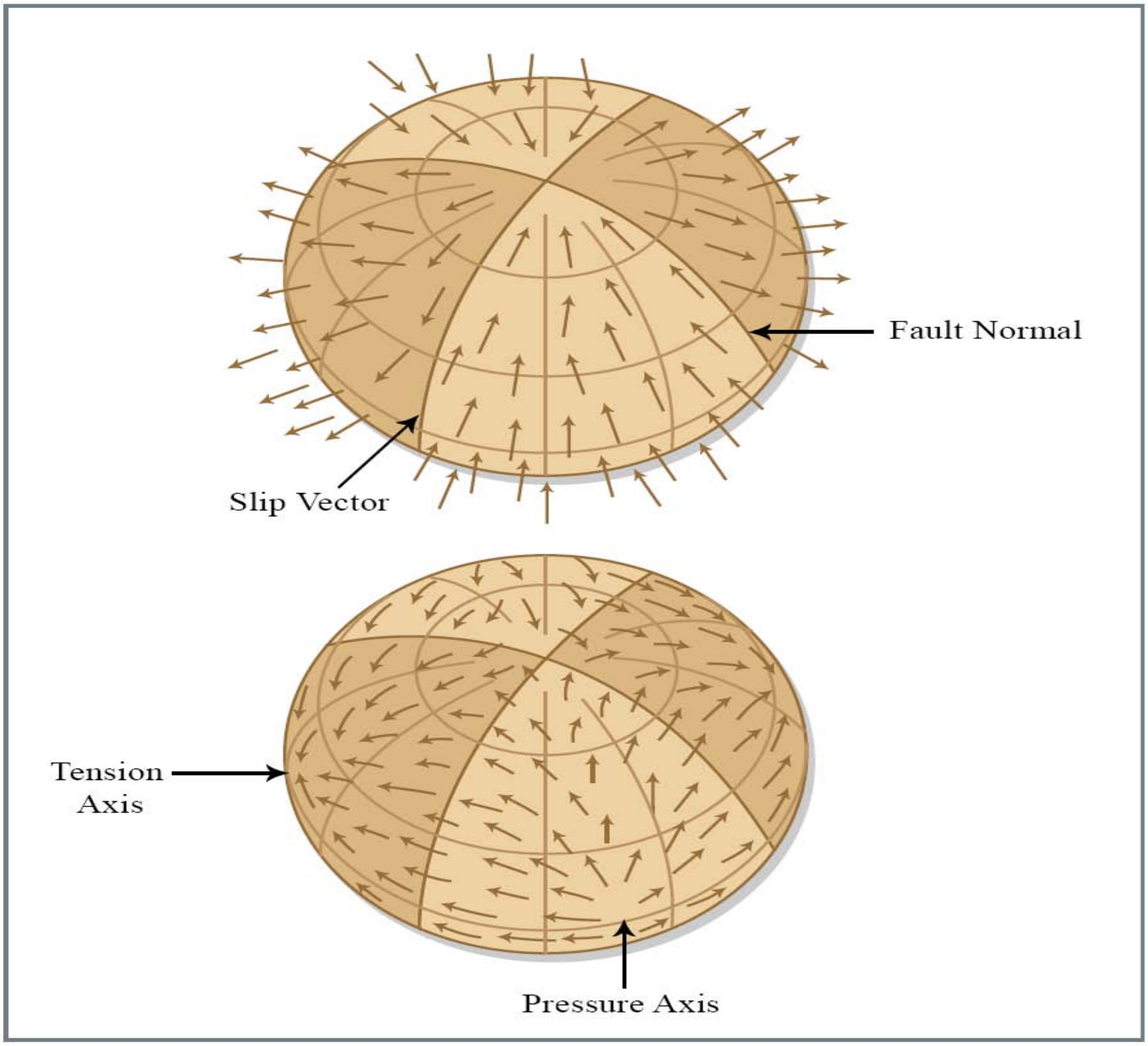


Figure by MIT OCW.

Figure 7: The far-field radiation pattern for P-waves (top) and S-waves (bottom) for a double-couple source (Source: Shearer, 1999).

Magnitude

Seismic Source ➔we have to find the location

Surface wave ➔ mechanisms

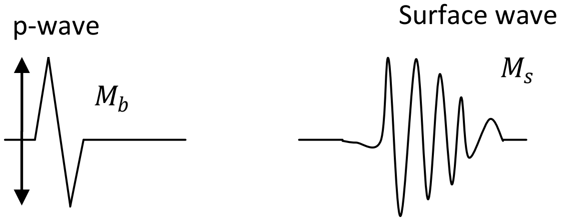


Figure 8: Magitude

Note: Please read Stein and Wysession, Chapter 4, for details.

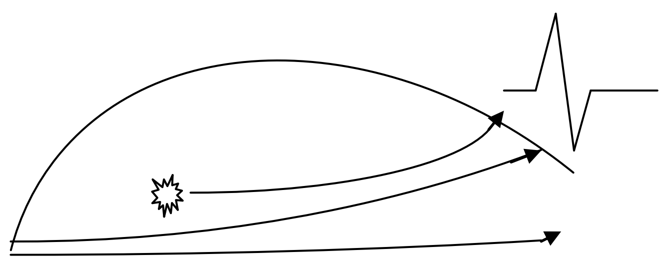


Figure 9

1. ➔ distance
2. ➔radiation pattern
3. ➔"Medium"
   a. Reflection/transmissions
   b. Anisotropy
   c. Heterogeneity
   d. Unelasticity (attenuation)

The first measure is the magnitude, which is based on the amplitude of the waves recorded on a seismogram. The wave amplitude reflects the earthquake size once the amplitudes are corrected for the decrease with distance due to geometric spreading and attenuation. Magnitude scales have the general form:

$$M = log\left(\frac{A}{T}\right) + f(h, \Delta) + C \tag{17}$$

where A is amplitude of the signal , T is dominant period, f is correction for the variation of amplitude with the earthquake's depth h, distance Δ from the seismometer, and C is the regional scale factor.

For global studies, the primary magnitude scales are:

The body wave magnitude m_b: measured from the early portion of the body wave train:

$$M_b = \log\left(\frac{A}{T}\right) + Q(h, \Delta) \tag{18}$$

Measurements of m_b depend on the seismometer used and the portion of the wave train measured. Common practice uses a period of ~1sec for the P and ~4s for the S.

The surface wave magnitude M_s: measured using the largest amplitude (zero to peak) of the surface waves

$$\begin{aligned} M_s &= log\left(\frac{A}{T}\right) + 1.661\, log\, \Delta + 3.3, \\ M_s &= log\, A_{20} + 1.661\, log\, \Delta + 2.0, \end{aligned} \tag{19}$$

where the first form is general and the second uses the amplitudes of Rayleigh waves with a period of 20 sec, which often have the largest amplitudes.

Limitations

These relations are **empirical** and thus no direct connection to the physics of earthquakes. Additionally, body and surface wave magnitudes do not correctly reflect the size of large earthquakes.

Magnitude saturation

It's a general phenomenon for M_b above about 6.2 and M_s above about 8.3.

Figure 10 shows the theoretical source spectra of surface and body waves. The two are identical below the ω^{-2} corner frequency. As the fault length increases, the seismic moment increases and the corner frequency moves to the left, to lower frequencies. The moment M_0 determining the zero-frequency level becomes larger. However, M_s, measured at 20 s, depends on the spectral amplitude at this period. For earthquakes with moments less than 10^{26} dyn-cm, a 20s period corresponds to the flat part of the spectrum, so M_s increases with moment. But for larger moments, 20s is to the right of the first corner frequency, so M_s does not increase as the same rate as the moment. Once the moment exceeds 5.10^{27}dyn-cm, 20 s is to the right of the second corner. Thus M_s saturates at about 8.2 even if the moment increases. It is similar for body wave magnitude, which depends on the amplitude at a period of 1s. Because this period is shorter that 20s, m_b saturates at a lower moment (~10^{25} dyn-cm), and remains at about 6 even for larger earthquakes.

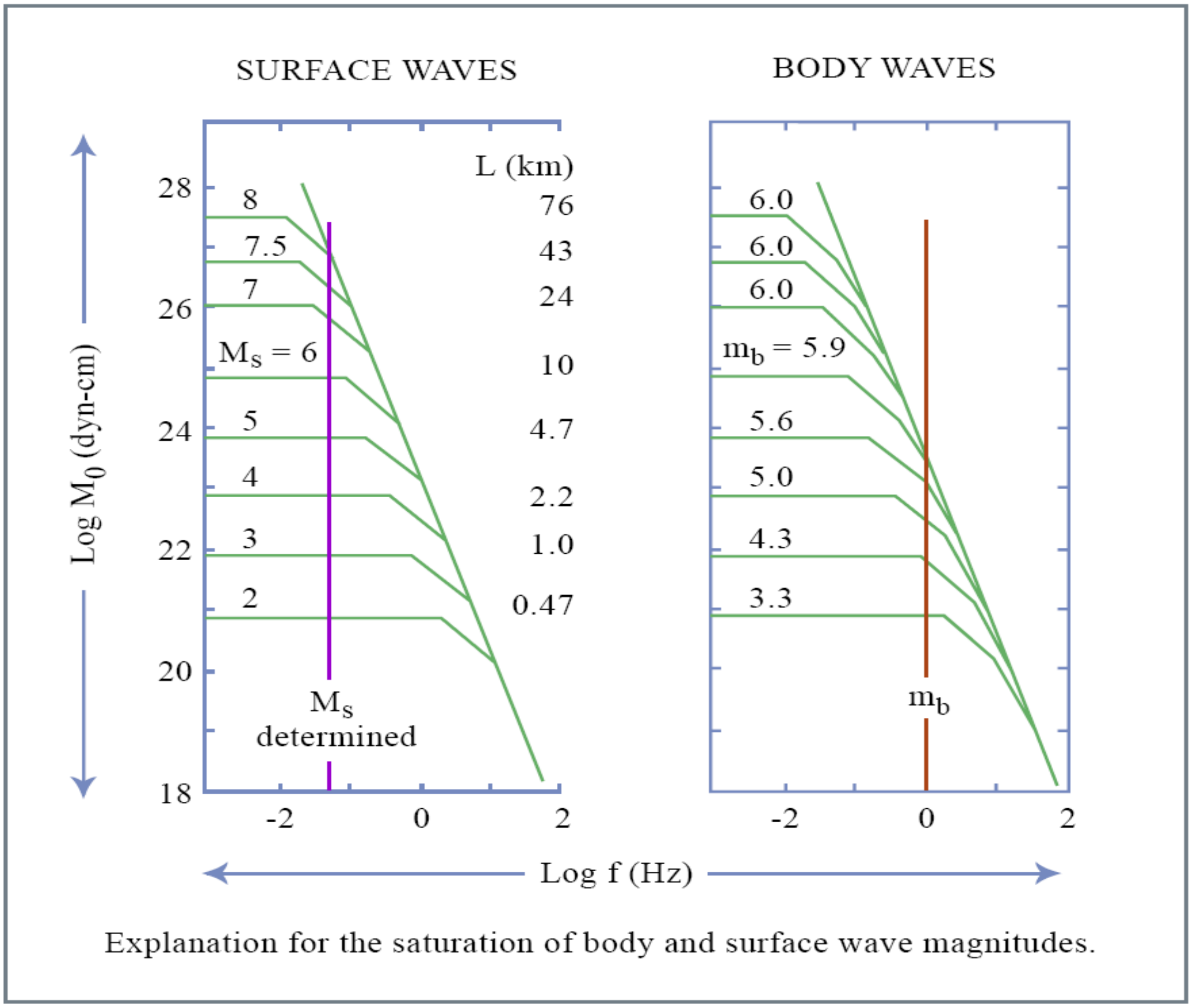


Figure by MIT OCW.

Figure 10: Saturated body and surface wave magnitudes (Source: Stein and Wysession, p221).

Moment magnitude

A simple solution by Kanamori (equation 21) defines the magnitude scale based on the seismic moment. The moment magnitude:

$$M_b = T \sim 1sec \qquad (20)$$
$$M_c = T \sim 20sec$$

$$M_w = \frac{logM_0}{1.5} - 10.73 \qquad (21)$$

This expression gives a magnitude directly tied to earthquake source processes that does not saturate. M_w is the common measure for large earthquakes. Estimation of M0 requires more analysis than for mb or Ms. However, semi-automated programs like the Harvard CMT project regularly compute moment magnitude for most earthquakes larger than Mw5 (http://www.seismology.harvard.edu/projects/CMT/).

An example of the Harvard CMT catalog:
010104J BALI REGION, INDONESIA
Date: 2004/ 1/ 1
Centroid Time: 20:59:33.6
GMT Lat= -8.45
Lon= 115.83
Depth= 35.6
Half duration= 2.0
Centroid time minus hypocenter time: 1.7
Moment Tensor: Expo=24 1.690 -2.190 0.503 2.530 1.590 5.520
Mw = 5.8
Mb = 5.5
Ms = 5.4
Scalar Moment = 6.58e+24
Fault plane: strike=349 **dip**=63 **slip**=162
Fault plane: strike=87 **dip**=74 **slip**=28

Updated by: Sami Alsaadan
Sources: notes from May 2, 2005, by Lori Eich.. May 4, 2005, Kang Hyeun Ji. May 9, 2005, by Sophie Michelet. May 2, 2008 lecture.
"An Introduction to Seismology, Earthquakes, And Earth Structure" by Stein &Wysession (2007).