1) (100%) Consider the homogeneous deformation:

\[ x' = ax + by \]
\[ y' = cx + dy \]

\[ x = \frac{(dx' - by')}{ad - bc} \]
\[ y = \frac{(ay' - cx')}{ad - bc} \]

a) Write expressions for the displacement vector \( \mathbf{u} \) that takes \((x,y) \rightarrow (x',y')\), both in terms of \((x,y)\) and \(x', y')\).

b) Write expressions for \( E_{ij} \) (Lagrangian strain tensor), \( e_{ij} \) (Eulerian strain tensor), \( \Omega_{ij} \) (Lagrangian rotation tensor), \( \omega_{ij} \) (Eulerian rotation tensor).

These can be applied incrementally over a time \( t \), \( 0 \leq t \leq 1 \), by setting

\[ a \rightarrow 1 + t(a - 1), \quad b \rightarrow bt, \quad c \rightarrow ct, \quad d \rightarrow 1 + t(d - 1). \]

c) Write expressions for \( e_{ij}(t) \) and \( \varepsilon_{ij}(t) \), where \( \varepsilon_{ij}(t) \) is the instantaneous Cauchy strain rate tensor.

Does

\[ e_{ij}(t = 1) = \int_{0}^{1} \dot{\varepsilon}_{ij}(t) dt \]

Why or why not?

Consider the 2 following finite strains:

1) \( x' = x + 1.5y \)
\( y' = y \)

2) \( x' = 2x \)
\( y' = y/2 \)

d) What special strains do they represent?

e) Write \( E_{ij}(t=1) \), \( e_{ij}(t=1) \), \( \Omega_{ij}(t=1) \), \( \omega_{ij}(t=1) \), and \( e_{ij}(t=1) \) for these cases.

f) Calculate the principal axes for \( E_{ij}(t=1) \), \( \dot{\varepsilon}_{ij}^e(t=1) \), and \( e_{ij}(t=1) \) for both cases.

g) In their final stage, pure and simple shears can be simply related by a rotation. Yet pure shear has 2 equivalent directions, while simple shear has only 1.

Materials such as ice or olivine develop preferred fabrics when subjected to simple shear. They recrystallize under deviatoric stress with easy glide at \( 45^\circ \) to the maximum compressive stress. Since the stresses for pure and simple shear are equivalent, how can pure shear lead to 2 preferred directions, while simple shear leads to only 1?