Strain production and preferred orientation

Strain during glide

- For n dislocations slipping:

\[ \varepsilon_{ij}^{total} = n \frac{b}{2h} \]

- For an increment:

\[ d\gamma = \frac{b}{h} \frac{dx}{l} \frac{t}{t} \]

\[ d\varepsilon = \frac{bda}{2V} \]
Inclined Slip Plane

- Strain
  - Burgers Vector
  - Normal to glide plane
  - Number of dislocations
Strain elements from glide

\[ e_{ij} \triangleq \frac{du_i}{dx_j} \equiv \epsilon_{ij} + \omega_{ij} \]

\[ |PP'| = \alpha h \beta = \alpha (r \cdot n) \beta \]

where \( \beta \) is a unit Burgers vector

and \( \alpha = \frac{s}{h} \)

\[ e_{11} = \frac{\partial}{\partial x_1} (\alpha (r \cdot n) \beta) = \alpha \frac{\partial}{\partial x_1} (x_k n_k) \beta_1 = \alpha n_1 \beta_1 = e_{11} \]
Strain and Rotation

\[ \varepsilon_{ij} = \alpha \frac{1}{2} \begin{pmatrix}
2n_1 \beta_1 & n_1 \beta_2 + n_2 \beta_1 & n_1 \beta_3 + n_3 \beta_1 \\
\cdot & 2n_2 \beta_2 & n_2 \beta_3 + n_3 \beta_2 \\
\cdot & \cdot & 2n_3 \beta_3
\end{pmatrix} \]

\[ \omega_{ij} = \alpha \frac{1}{2} \begin{pmatrix}
0 & n_2 \beta_1 - n_1 \beta_2 & n_3 \beta_1 - n_1 \beta_3 \\
\cdot & 0 & n_3 \beta_2 - n_2 \beta_3 \\
\cdot & \cdot & 0
\end{pmatrix} \]

- No component of \( \beta \) or \( n \) in \( k \) direction \( \rightarrow \varepsilon_{ik} = 0 \)
- No climb or diffusion
  \( -n_1 \beta_1 = n_2 \beta_2 + n_3 \beta_3 \)
- Rotation (and strain) depend on activity (\( \alpha \))
Independent Slip Systems

- Distinct systems can give rise to same strain. (e.g. interchange n and β)
- If strain element unique, then independent.
- No more than two β’s on the same plane can be independent.
- Crystallographic symmetry can increase number of strain elements for a particular slip systems.
Strain from climb

- For climb only
  \[ \gamma = \frac{S}{l} \quad u = \gamma (r \cdot \beta) \beta \]
  \[ e_{ij} = \frac{\partial}{\partial x_j} (\gamma (r \cdot \beta) \beta) = \gamma \frac{\partial}{\partial x_j} (x_k \beta_k) \beta_j = \gamma (\beta_i) \beta_j \]
  \[ \varepsilon_{ij} = \frac{\gamma}{2} (\beta_i \beta_j + \beta_j \beta_i) = \gamma \beta_i \beta_j \]

- Strain is irrotational
- Depends only on \( \beta \) not \( n \).
- Open system, so 6 ind. s.s.
- Three \( \beta \)'s climbing and gliding give 6 systems.
Taylor-von Mises Criterion

- Low T, glide easier than climb. Dilatancy may result.
- For homogeneous, non-dilatant, creep, 5 independent slip systems must be present.
- If dilatant, 6 independent slip systems necessary.
- If condition not fulfilled
  - twinning
  - climb or diffusion
  - void production
  - inhomogeneous flow
Independence of Slip Systems

- Convert vectors to Cartesian system. Choose 5 easiest systems.
- If no climb allowed, express strain as 5-dimensional vector. \([\dot{\varepsilon}_{11} - \dot{\varepsilon}_{22}, \dot{\varepsilon}_{33} - \dot{\varepsilon}_{22}, \dot{\varepsilon}_{12}, \dot{\varepsilon}_{23}, \dot{\varepsilon}_{13}]\)
- Form 5x5 matrix, take determinant

\[
\begin{vmatrix}
(\varepsilon_{11} - \varepsilon_{33})' & (\varepsilon_{11} - \varepsilon_{33})'' & (\varepsilon_{11} - \varepsilon_{33})''' & (\varepsilon_{11} - \varepsilon_{33})'''' & (\varepsilon_{11} - \varepsilon_{33})'''''
(\varepsilon_{22} - \varepsilon_{33})' & (\varepsilon_{22} - \varepsilon_{33})'' & (\varepsilon_{22} - \varepsilon_{33})''' & (\varepsilon_{22} - \varepsilon_{33})'''' & (\varepsilon_{22} - \varepsilon_{33})'''''
\dot{\varepsilon}_{12} & \dot{\varepsilon}_{12}'' & \dot{\varepsilon}_{12}''' & \dot{\varepsilon}_{12}'''' & \dot{\varepsilon}_{12}'''''
\dot{\varepsilon}_{23} & \dot{\varepsilon}_{23}'' & \dot{\varepsilon}_{23}''' & \dot{\varepsilon}_{23}'''' & \dot{\varepsilon}_{23}'''''
\dot{\varepsilon}_{13} & \dot{\varepsilon}_{13}'' & \dot{\varepsilon}_{13}''' & \dot{\varepsilon}_{13}'''' & \dot{\varepsilon}_{13}'''''
\end{vmatrix}
= 0
\]
Deformation of Polycrystals

- If 5 independent ss’s available, homogenous, non-dilatant flow possible.
- If inhomogeneous flow possible, then 4 ss’s sufficient.
- If dilatancy required, flow is pressure dependent.
- With only two ss’s, impossible to get pressure independent flow.
  - Basal slip, e.g. mica.
Texture, Fabric, and Preferred Orientation

- **Texture**: Geometrical aspects of component particles of a rock, including size, shape, and arrangement.

- **Fabric**: Orientation in space of elements of which rock is composed. That factor of the texture which depends on the relative sizes and shapes, and the arrangement of the component crystals.

- **Preferred orientation**: A rock in which the grains are more or less systematically oriented by shape or [by crystallographic orientation].

Methods of measuring

- Optical
- X-ray pole figure goniometer
- Synchrotron X-rays
- Neutron diffraction
- TEM
- EBSD (EBSP)
Data representation

- Pole figures: Density distribution of a single pole plotted in a stereographic plot relative to the sample coordinates.
- ODF: An orientation probability distribution function of three Euler angles
Simulations

- **Taylor- equi-strain**
  - \( \cong \) Voight elastic bound
  - fcc bcc metals (hi symm.)
  - upper bound in strength

- **Equi-stress-Sachs**
  - \( \cong \) Voight elastic bound
  - lower bound
  - heterogeneous strain

- **Self-consistent (VPSC)**

- **Finite element**
Processes

- Constitutive law
- Grain growth/Recrystallization
- Metamorphic reactions
- Dilatancy