1. Consider the following linearized equations in 1 and 3 dimensions and impose a wave solution as shown:

<table>
<thead>
<tr>
<th>Linearized Equation</th>
<th>Plane wave</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_t + c \phi_x = 0 )</td>
<td>( e^{i k x - i \omega t} )</td>
</tr>
<tr>
<td>( \phi_{tt} - c^2 \phi_{xx} = 0 )</td>
<td>( e^{i k x - i \omega t} )</td>
</tr>
<tr>
<td>( \phi_t + \bar{c} \nabla \phi = 0 )</td>
<td>( e^{i k \cdot \bar{x} - i \omega t} )</td>
</tr>
<tr>
<td>( \phi_{tt} - c^2 \nabla^2 \phi = 0 )</td>
<td>( e^{i k \cdot \bar{x} - i \omega t} )</td>
</tr>
<tr>
<td>( \nabla^2 \phi_t + \beta \phi_x = 0 )</td>
<td>( e^{i k \cdot \bar{x} - i \omega t} )</td>
</tr>
</tbody>
</table>

Find the dispersion relation for each of them and the phase speed (or three phase speed in 3 dimensions)

2. Suppose a wave is found that has the form

\[ \phi = A e^{i \theta} \]

where

\[ \theta = -\alpha t^2 / x \]

a) If the wave can be thought of as slowly varying, what are its frequency, wavenumber and dispersion relation?

b) Now that you have \( \omega \) and \( k \), when will the slowly varying assumption be valid?

3. Consider the interface between two semi-infinite fluids of different densities:
Imposing wave solutions for \((\phi_1, \phi_2, \eta)\) that obey the deep water equations derive the boundary conditions that must be satisfied at \(z = 0\) and the dispersion relation. What type of wave have you obtained?

4. Consider a deep water wave impinging on a current \(V(x)\) of the following shape:

\[
\begin{align*}
\text{a)} & \quad \text{What is the dispersion relation?} \\
\text{b)} & \quad \text{Derive the ray equations for } (\omega, k, \ell) \text{ and discuss their implications.} \\
\text{c)} & \quad \text{A wave packet starts its motion with initial conditions } (\ell_0, k_0, \omega_0) \text{ where } V(x) \equiv 0 \text{ and impinges on the current. What is } k(x) \text{? Sketch the variation of the wave from } x = 0 \text{ to } x = L. \\
\text{d)} & \quad \text{If the wave ray moves as in the sketch:}
\end{align*}
\]

\[
\begin{align*}
\text{What is } \sin \theta(x) \text{? What is the ratio } \frac{\sin \theta(x)}{\sin \theta_0}?
\end{align*}
\]