12.804 — Baroclinic Inversion/ Instability
— Numerical Experiments

This is a two layer version of the doubly–periodic, quasigeostrophic code we used to study Rossby waves and vortices. It solves the equations

\[
\begin{aligned}
\left[ \frac{\partial}{\partial t} + U_1 \frac{\partial}{\partial x} + J(\psi_1, \cdot) \right] q_1 + [\beta + F_1(U_1 - U_2)] \frac{\partial}{\partial x} \psi_1 &= \text{filter} \\
\left[ \frac{\partial}{\partial t} + U_2 \frac{\partial}{\partial x} + J(\psi_2, \cdot) \right] q_2 + [\beta + F_2(U_2 - U_1)] \frac{\partial}{\partial x} \psi_2 &= \text{filter}
\end{aligned}
\]

with the inversion formulae

\[
\begin{aligned}
q_1 &= (\nabla^2 - F_1)\psi_1 + F_1\psi_2 \\
q_2 &= (\nabla^2 - F_2)\psi_2 + F_2\psi_1
\end{aligned}
\]

The model runs via http://puddle/-glenn/12.804 with the appropriate link on the Linux machines. For the inversion, you specify the parameters \(U_1, U_2, F_1, F_2,\) and \(\beta.\) Given the fields for \(q_1\) and \(q_2\) as functions of \(x\) and \(y,\) the program will calculate \(\psi\) and contour both the PV anomalies \(q_i\) and the full PV fields \(q_i + [\beta + F_i(U_i - U_{3-i})]y.\) It will also show the streamfunction anomalies \(\psi_i\) and the full streamfunction \(\psi_i - U_i y.\)

Once you have specified the PV and/or streamfunction fields, use QG model to see how the flow evolves. The parameters are similar to those in the BT vorticity equation solver.

Experiments to consider

- Explore the relationship between upper layer PV anomalies and the flows in both layers.
- Explore the instability criterion.
- Show that stable waves can still amplify, at least temporarily, if the initial phase relationships between upper and lower layers are correct.
- Examine the interaction of two blobs of anomalous PV, one upper layer and one lower. Figure out the conditions under which they will reinforce each other. (Hint — remember that the primary effect of the PV anomalies in linear theory is to advect the background PV gradients.) What happens in the nonlinear regime?
- A growing plane wave is an exact solution to the equations above. What happens when such a wave is perturbed? Compare unperturbed to perturbed solutions.