Consider a spherical earth of radius $R=6.37 \times 10^6$ meters, rotating at a rate of $\Omega=7.29 \times 10^{-5}$ s$^{-1}$. Starting from rest at point 1 on the surface of the earth (latitude of $15^o$N), a particle is moved to point 2 near the equator ($5^o$N). It is then moved upward a distance $\Delta R=10$ km to point 3, then to point 4 at a latitude of $25^o$N, then downward to the surface to point 5. In all moves, total angular momentum, $\omega$ is conserved. For each point, what is the zonal velocity, $u$, relative to the rotating earth? Do you find reasonable values for the easterly trades or the westerly ‘jetstream’? If not, what might be wrong?

$$\omega = mr \left( u + \Omega r \right),$$

where $m$ is the mass of the particle, $u$ is the zonal velocity relative to the earth, $\theta$ is the latitude, & $r$ the distance from the earth’s rotation axis: $r = R \cos \theta$. 

Note, this is not drawn to scale: radial distances are highly exaggerated.
One aspect of the density of seawater we want to explore here is how density varies with different temperatures and salinities (at zero or atmospheric pressure). This is shown in the upper panel. Note that the temperature of maximum density for each salinity curve, $T_m$, decreases with increasing salinity. This is shown in the lower panel (see red x’s). Also plotted is the decrease in the freezing point of seawater with increasing salinity (blue o’s). Note that the two different curves cross over at a point $(T,S) = (-1.33, 24.695)$.

Problem: Consider a freshwater ($S=0$) pond and a saltwater ($S=35$) pond of equal depths, say 50m. In winter under cold, calm conditions, which pond will form ice first? Why? When both ponds begin to form ice, what is the bottom temperature in each? (Hint: you might have to consider both curves in the lower panel for the correct answer.)
12.808, Problem 3, problem set #1

Consider the problem illustrated by the Matlab program Pucks_on_ice. In order to get this m-file, you should download both the main program pucks_on_ice.m as well as the files draw_fig2.m and deriv.m.

Try running the program. In the explanation box describing the figures (Figure(6), there are some suggestions. Try the following, in each case showing plots of the results and your interpretation of why trajectories may be different from the initial example:

1. Change the bump to a dimple, making \( h_0 = -0.05 \) (meters) in pucks_on_ice.m, line 128
2. For the bump, change the latitude (line 129) to the north pole (\( \text{lat} = 90 \))
3. For the bump, change the latitude to the equator (\( \text{lat} = 0 \))
4. For the bump, change the latitude of the bump to \(-30\) (southern hemisphere)
5. For PO students (others optional), consider the equations with no body or topography forces. Derive expressions for the evolution of the oscillation for small friction and determine frequency of the oscillation. What are the equations representing the geostrophic balance and why do the particles rotating around the bump slowly spiral out? This is related to an example of another type of ekman layer: due drag of a moving particle by a solid surface (e.g. the ocean bottom).