Exercises

11.1 Using scale analysis, determine under what conditions the $\beta$-plane is an acceptable approximation to the rotating sphere.

11.2 Assess the accuracy of geostrophic balance for Rossby waves.

11.3 For the parameters used in connection with stationary waves (i.e., the values of $d, n = 0, k, f$, and $\beta$ — and taking $h = 10.4$ km) find that value of $k$ for which $\sigma$ (for Rossby waves) is a maximum.

11.4 Write down the horizontal momentum equations for perturbations $u', v', p'$ in a form applicable to an equational $\beta$-plane, that is, with $f = \beta y$. Assume a solution with $v' = 0$, and

$$u' = \text{Re}\{u'_0 \exp (iwt - kx)\}.$$  

Show that in this case the amplitude $u'_0$ varies with the latitude as

$$\exp \left( \frac{-\beta y^2 k}{2\omega} \right).$$

Note that for a satisfactory solution $k$ must be positive, that is, the wave must be eastward moving. Plot the pressure field associated with the variation in zonal velocity $u'$. These are equatorial Kelvin waves; they have been observed in the stratosphere.