Time-dependent, axisymmetric model phrased in R space

- Hydrostatic and gradient balance above PBL
- Moist adiabatic lapse rates on M surfaces above PBL
- Boundary layer quasi-equilibrium
- Deformation-based radial diffusion
Potential Radius:

\[ \frac{f}{2} R^2 \equiv M = rV + \frac{f}{2} r^2 \]  

(1)

Local energy conservation:

\[ \frac{1}{r_b^2} - \frac{1}{r_t^2} = \frac{-2(T_s - T_t)}{f^2 R^3} \frac{\partial s^*}{\partial R} \]  

(2)

Differentiate in time:

\[ \frac{1}{r_b^3} \frac{\partial r_b}{\partial \tau} - \frac{1}{r_t^3} \frac{\partial r_t}{\partial \tau} = \frac{(T_s - T_t)}{f^2 R^3} \frac{\partial \partial R \partial s^*}{\partial \tau} \]  

(3)
Mass continuity:

\[ ru = \frac{\partial \psi}{\partial p}, \]

\[ r \omega = -\frac{\partial \psi}{\partial r} \]

Transform to potential radius coordinates:

\[ ru = r \frac{dr}{dt} = r \left[ \frac{\partial r}{\partial \tau} + \frac{\partial r}{\partial R} \frac{dR}{dt} + \frac{\partial r}{\partial P} \frac{dP}{dt} \right] = \frac{\partial \psi}{\partial p}, \]

\[ \rightarrow r \frac{\partial r}{\partial \tau} = \frac{\partial \psi}{\partial p} - r \omega \frac{\partial r}{\partial P} = \frac{\partial \psi}{\partial p} + \frac{\partial \psi}{\partial r} \frac{\partial r}{\partial P} = \frac{\partial \psi}{\partial P} \]
Define $\psi_0$ as streamfunction at top of boundary layer and use simple finite difference in vertical:

\[
\frac{\partial r_b^2}{\partial \tau} \approx 2(\psi_0 - \psi),
\]

\[
\frac{\partial r_t^2}{\partial \tau} \approx 2\psi
\]  

(4)
PBL flow:

\[
\frac{\partial r}{\partial \tau} + r \frac{\partial r}{\partial R} \frac{dR}{dt} = \frac{\partial \psi}{\partial P}
\]

Angular momentum balance:

\[
V = \frac{f}{2} \frac{R^2 - r^2}{r}
\]

\[
\frac{f}{2} \frac{dR^2}{dt} = -gr \frac{\partial \tau_\theta}{\partial P}
\]

\[
\rightarrow r \frac{\partial r^2}{\partial R^2} \frac{2}{f} g \frac{\partial \tau_\theta}{\partial P} \approx - \frac{\partial \psi}{\partial P},
\]

\[
\psi_0 = -\frac{2}{f} g \frac{\partial r^2}{\partial R^2} \tau_s = \frac{2}{f} g \frac{\partial r^2}{\partial R^2} \rho_s C_D |V| V
\]
Saturation entropy:

\[ \frac{\partial s^*}{\partial \tau} = \Gamma_d \frac{\Gamma}{\Gamma_m} \left[ \left( M_u - M_d - w \right) \frac{\partial s_d}{\partial z} + \frac{\dot{Q}_{\text{rad}}}{T} \right] \]

Downdraft:

\[ M_d = \left( 1 - \varepsilon_p \right) M_u \]

Boundary layer entropy:

\[ h_s \frac{\partial s}{\partial \tau} = C_k |V| \left( s_0^* - s \right) + C_D |V|^3 - \left( M_u - w_0 \right) \left( s - s_m \right) + C_D r |V| V \frac{\partial s}{fR\partial R} \]

*Used to define \( M_{ueq} \) when \( > 0 \); otherwise, equation integrated for \( s \)
Relaxation equation:

\[ \frac{\partial M_u}{\partial \tau} = \frac{M_{ueq} - M_u}{\tau_c} \]

Precipitation efficiency:

\[ \varepsilon_p = \frac{S_m - S_{m0}}{S - S_{m0}} \]

Middle troposphere entropy:

\[ \frac{\partial S_m}{\partial \tau} = \Lambda M_u \left( s - s_m \right) + \dot{Q}_{rad} \]
Radiation: \[ \dot{Q}_{rad} \approx -(s^* - s^*(t = 0)) \]

Radial diffusion added to equations for \( r_b, r_t, s^*, \) and \( s_m \)

\[
D_b = -\frac{1}{R} \frac{\partial}{\partial R} \left[ r_b^2 \nu_b \frac{\partial}{\partial r_b} \left( \frac{R^2}{r_b^2} \right) \right]
\]

\[
D_t = -\frac{1}{R} \frac{\partial}{\partial R} \left[ r_t^2 \nu_t \frac{\partial}{\partial r_t} \left( \frac{R^2}{r_t^2} \right) \right]
\]

\[
D_{s^*} = \frac{\partial}{\partial r_b^2} \left( r_b \nu_b \frac{\partial s^*}{\partial r_b} \right)
\]

\[
D_{s_m} = \frac{\partial}{\partial r_m^2} \left( r_m \nu_b \frac{\partial s_m}{\partial r_m} \right)
\]
\[ v_i = l^2 \left| r_i \frac{\partial}{\partial r_i} \left( \frac{R^2}{r_i^2} \right) \right| \]

Note that surface saturation entropy depends on pressure, which is calculated from gradient wind balance using \( V \).

Complete equations summarized in Emanuel (1995), posted on course web page.
Model behavior

![Graph showing model behavior with time on the x-axis and maximum surface wind speed on the y-axis. The graph includes three lines: blue for Control, green dotted for Theory, and red for Weak initial vortex. The x-axis ranges from 0 to 60 days, and the y-axis ranges from 0 to 80 m/s. The graph shows a rapid increase in wind speed for the Control line, while the other lines remain relatively flat.]
Saturate troposphere inside 100 km in initial state:

![Graph showing maximum surface wind speed over time for different conditions.](image)

- **Control**
- **Theory**
- **Saturated core**

Time (days) vs. Maximum surface wind speed (m/s)
Character of control simulation

Azimuthal velocity, from 0 to 68.0355 m/s
Cumulus mass flux, from 0 to 18.1277 m/s
Radial velocity, from -27.7593 to 74.5129 m/s
Vertical velocity, from -0.2099 to 19.6568 m/s (- values X 10)
Equivalent potential temperature, from 329.8344 to 368.7422 K
Perturbation pressure, from -61.4327 to 2.5845 mb
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