Final homework assignment

Problem 1. Temperature and salinity eddy fluxes.

The tracer conservation equation is

\[
\frac{\partial Q}{\partial t} + \nabla \cdot (uQ) = \kappa_Q \nabla^2 Q
\]  

(1)

where \( Q \) is any tracer, and \( \kappa_Q \) is the molecular diffusivity of that tracer.

(a) Assuming that \( Q \) can be split into a large scale component \( \overline{Q} \) and a small scale component \( Q' \), so that \( Q = \overline{Q} + Q' \) and similarly for the velocity field, write down equations for the time-evolution of the large scale temperature \( T \) and salinity \( S \).

(b) Assume that the small-scale dynamics influence the large scale fields only through vertical fluxes, which can be parameterized in terms of diffusion down the large scale gradient with eddy diffusivity \( \kappa_T^* \) for temperature and \( \kappa_S^* \) for salinity. Rewrite the equations from (a) incorporating this parameterization of the small-scale fluxes.

(c) Now combine both equations from (b) to form one equation for the large scale density \( \overline{\rho} = \beta S - \alpha T \), and show that the small scale fluxes of density can again be written in terms of diffusion in the direction of the large scale density gradient, with eddy diffusivity

\[
\kappa_p^* = \frac{\kappa_T^* R_p - \kappa_S^*}{R_p - 1}
\]  

(2)

where \( R_p = \frac{\beta \partial T / \partial z}{\alpha \partial S / \partial z} \).

(d) For warm salty water overlying cold fresh water, small-scale fluxes of salt and temperature may be due to either turbulent mixing, or salt-fingering. In the turbulent regime, salt and heat are mixed equally efficiently. In the salt fingering regime \( (\alpha w^T T)/(\beta w^S S) \approx 0.7 \). Find \( \kappa_p^*/\kappa_T^* \) under these two circumstances. Comment on the sign of \( \kappa_p^* \).

Problem 2. Stokes drift.

a) Surface gravity waves have \( \psi \approx \cos k(x - ct) \exp(kz) \). Find the particle trajectories at lowest order

\[
\frac{\partial}{\partial t} (X - x_0) = u(x_0, t)
\]

and then write the next order equations and find the mean displacement over a wave period.
b) Suppose the particle is constrained to a fixed depth $z_0$. What is its drift then? What if it’s a fixed distance below the free surface?