1. Show that the weighted/tapered least-squares objective function with Lagrange multipliers (Eq. 2.169 of the notes), produces the same solution as objective function Eq. (2.122) of the notes. (Eq. 2.173 of the notes should read $\tilde{\mu} = W^{-1}n$.)

2. (a) You have three equations in two unknown $x_i$,

\[
\begin{align*}
&x_1 + x_2 + n_1 = 1, \\
&x_1 - x_2 + n_2 = 2, \\
&x_1 + x_2 + n_3 = 1.5.
\end{align*}
\]

There is reason to believe that $\langle n_i \rangle = 0$, $\langle n_1^2 \rangle = \langle n_2^2 \rangle = 1$, $\langle n_3^2 \rangle = 16$. Find a defensible estimate of $x_i, n_i$, explaining what you are doing (don’t just give me numbers).

(b) Suppose now, in addition, that it is thought that $\langle x_1^2 \rangle = 10 \langle x_2^2 \rangle$, $\langle x_1 x_2 \rangle = 3$. What is a new solution using this information?

3. Using an eigenvector/eigenvalue analysis, solve (a)

\[
\begin{bmatrix}
1 & -1 & -2 \\
-1 & 2 & -1 \\
-2 & -1 & 6
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} =
\begin{bmatrix}
1 \\
0 \\
1
\end{bmatrix},
\]

and (b)

\[
\begin{bmatrix}
1 & -1 & -2 \\
-1 & 2 & -1 \\
1.5 & 2 & -2.5
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} =
\begin{bmatrix}
1 \\
0 \\
1
\end{bmatrix}
\]