1 Production Possibilities Frontier

Marginal rate of transformation (\(MRT\)):

- How much clothing must be given up to produce one additional unit of food.
- The absolute value of the slope of the production possibilities frontier.
- If \(MRT\) increases in food, then the production possibilities frontier is concave.

\[
MRT = \frac{MC_F}{MC_C}.
\]

Proof. Reducing \$1 input from clothing, \(C\) decreases by \(\frac{1}{MC_C}\); adding \$1 input to food, \(F\) increases by \(\frac{1}{MC_F}\). Thus,

\[
MRT = \frac{\Delta C}{\Delta F} = \frac{MC_F}{MC_C} = \frac{MC_F}{MC_C}.
\]

2 Output Market Efficiency

Suppose we have two industries, clothing and food, in the market. Consumers have demand for the two goods. They have a representative utility \(U(C, F)\). A Pareto efficient result occurs when the production possibilities frontier is tangent to the indifference curve (see Figure 3). That is to say,

\[
MRT = MRS.
\]
2 Output Market Efficiency

Figure 1: Production Contract Curve.

Figure 2: Production Possibilities Frontier.
2 Output Market Efficiency

Figure 3: Production Possibilities Frontier and Indifference Curve.

Figure 4: Equilibrium in the Output Market.
2.1 General equilibrium in the output market

The prices are $P_F$ for food, and $P_C$ for clothing. When the market reaches its equilibrium, industries are maximizing their profits, so

$$MC_F(q) = P_F;$$
$$MC_C(q) = P_C.$$ 

Thus,

$$MRT = \frac{MC_F}{MC_C} = \frac{P_F}{P_C}.$$ 

Consumers maximize their utility, so

$$MRS = \frac{P_F}{P_C}.$$ 

Combining the equations together, we obtain (see Figure 4)

$$MRT = \frac{P_F}{P_C} = MRS.$$ 

Consider non-equilibrium prices $P'_F$ and $P'_C$,

$$\frac{P'_F}{P'_C} < \frac{P_F}{P_C}.$$ 

Given the prices, food has a shortage and clothing has an excess (see Figure 5). The prices will change to adjust to the equilibrium state, namely, $P'_F$ increases and $P'_C$ decreases.

2.1 General equilibrium in the output market

Example (Gains from Free Trade). Assume that Holland and Italy both produce cheese and wine, unit of labor required is provided in Table 2.1. If these two countries cannot trade cheese or wine, we consider the domestic markets separately. The price ratio is not the same:

$$\frac{P'_W}{P'_C} < \frac{P_C}{P_C}.$$ 

Consumer utility levels are $U_H$ and $U_I$, respectively. However, if they can trade, Holland exports cheese and imports wine, and Italy exports wine and imports cheese. The prices ratio will adjust to agree, and people in both countries are better off because both indifference curves move upwards (see Figure 6). The new utility levels are $U'_H$ and $U'_I$.

<table>
<thead>
<tr>
<th></th>
<th>Cheese</th>
<th>Wine</th>
</tr>
</thead>
<tbody>
<tr>
<td>Holland</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Italy</td>
<td>6</td>
<td>3</td>
</tr>
</tbody>
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Table 1: Unit of Labor Required in Cheese and Wine Production.
2.1 General equilibrium in the output market

Figure 5: Non-equilibrium Consumption and Production.
2.1 General equilibrium in the output market

(a) Trade in Holland.

(b) Trade in Italy.

Figure 6: Gains from Free Trade.