1 Production and Costs

1.1 Lecture 8- Production Theory

1.1.1 Production Function

1. \( q = f(L, K) \)
   (a) \( q \) = units of output
   (b) \( L, K \) = labor and capital inputs

2. Marginal Product
   (a) The additional output gained from one extra unit of an input, holding the other inputs constant
   (b) **Marginal Product of Labor**- The additional output gained from one extra unit of labor, holding the other inputs constant
      i. \( MP_L = \frac{\partial q}{\partial L} \)
   (c) **Marginal Product of Capital** The additional output gained from one extra unit of labor, holding the other inputs constant
      i. \( MP_K = \frac{\partial q}{\partial K} \)

3. Isoquants
   • Slices of the production function that show combinations of \( K \) and \( L \) that produces the same level of output \( q \)
     - Isoquants are the analogues of indifference curves
     - Their shape is determined by the substitutability between \( K \) and \( L \)
   • The slope is called the **Marginal Rate of Technical Substitution (MRTS)**
     \[ MRTS = -\frac{MP_L}{MP_K} = -\frac{\partial q}{\partial K} \]
     - The isoquant exhibits diminishing margins- each additional unit of labor (capital) increases \( q \) less than the previous unit and it worth less in terms of forgone capital (labor)
     - Math side note for how to derive MRTS
       * Take the total derivative of our production function to see how total output is changing with respect to changes in inputs
         1. \( q = f(L, K) \)
         2. \( dq = \frac{\partial q}{\partial L} * dL + \frac{\partial q}{\partial K} * dK \)
     1. To produce an isoquant, we want to take a slice of the production function such that output is not changing \( (dq = 0) \)
     2. What is the slope for the isoquant? Just rearrange the terms
        (a) \( 0 = \frac{\partial q}{\partial L} * dL + \frac{\partial q}{\partial K} * dK \)
        (b) \( -\frac{\partial q}{\partial L} * dL = \frac{\partial q}{\partial K} * dK \)
        (c) \( \frac{dL}{dK} = -\frac{\partial q}{\partial K} = MRTS \)
1.1.2 Short Run Production
- At least one input is fixed
- For the purposes of our course, we will assume that capital (K) is fixed in the short run and that labor is variable

1.1.3 Long Run Production
- All inputs are variable- Firms can fully decide how much capital (K) and labor (L) to hire

1.1.4 Returns to Scale
1. Constant- \( f(2L, 2K) = 2f(L, K) \)
2. Decreasing- \( f(2L, 2K) < 2f(L, K) \)
3. Increasing- \( f(2L, 2K) > 2f(L, K) \)

1.1.5 Productivity

1.2 Lecture 9- Production and Costs
1.2.1 Short Run Costs
1. Fixed Costs are the costs of inputs that can’t be varied in the short run
   (a) In this course this is capital
2. Variable Costs are the costs of inputs that can be varied in the short run
   (a) In this course this is labor
3. Total Costs are the sum of fixed and variable costs: \( C = F + VC \)
4. Marginal Cost is the extra cost for another unit of output:
   (a) \( MC = \frac{dC}{dq} \) where \( C \) is the total cost
   (b) In the short run- \( MC = \frac{dVC}{dq} \) - the marginal cost is determined by the increase in the variable cost (since fixed costs do not vary with output)
5. Average Cost the average cost of production per unit produced.
   (a) \( AC = \frac{C}{q} \)
   (b) \( AVC = \frac{VC}{q} \) Average variable costs
   (c) \( AFC = \frac{FC}{q} \) Average fixed costs

1.2.2 Long Run Costs
- In the very long run, all input costs are variable - so choice is over input mix to maximizes production efficiency, or minimizes costs
1.2.3 How much to produce?
To tackle this question, you want to divide your takes into two parts

1. Derive the cost function (short or long run)
   (a) Derive a cost function that produces a given quantity of q most efficiently by combining K and L for a given set of factor prices w and r

2. Choose a quantity that maximizes profits for a given price

1.2.4 Deriving the Cost function
The question we are trying to answer is for a given unit of q, what is the cheapest way for me to combine K and L

1. We start by defining the analogue of the budget constraint- the isocost line
   (a) Isocost Line- combinations of labor and capital that produce the same level of cost
      i. \( C = w \cdot L + r \cdot K \)

2. Like in consumer optimization, we want to produce at the tangency point between the isocost and the isoquant. In other words, we want to find the lowest isoquant for a given isoquant
   (a) \( MRTS = -\frac{MP_L}{MP_K} = \frac{w}{r} \)

3. Deriving the total cost curve
   (a) Use your production function and point (2) to derive relationships between inputs and q
   (b) Plug these relationships back into the total cost function \( C = w \cdot L + r \cdot K \)

Remember that in the short run, capital is fixed so your cost function should reflect this

1.2.5 Long run average cost vs short run average cost curves
the LRAC is the lower envelope of the SRAC for different plant sizes. The LR cost of production is lower than the SR cost of production

1.2.6 Economies of Scope

1.3 Lecture 10- Competition

1.3.1 Perfect Competition
Firms and consumers are price takers. Why?

1. many small buyers and sellers
2. identical products
3. symmetric information
4. no transactions costs
5. free exit and entry in the long run
1.3.2 Residual Demand

Does the fact that a firm is a price taker (ie the demand curve they face is perfectly elastic) mean that the market demand is perfectly elastic? No. The residual demand (or the demand curve a firm faces) is much more elastic than the overall demand curve of the market.

1.3.3 Profit maximization

This refers to the second step in from 1.2.3. The firm wants to maximize profits, defined as

\[ \pi = R(q) - C(q) \]

where \( R(q) \) are the total revenues the firm receives from selling output \( q \) and \( C(q) \) is the cost we derived in 1.2.4

A firm therefore solves the following problem

\[ \frac{\partial \pi(q)}{\partial q} = \frac{\partial R(q)}{\partial q} - \frac{\partial C(q)}{\partial q} \]

\( \frac{\partial R(q)}{\partial q} = \frac{\partial C(q)}{\partial q} \) (want the point at which profits are maxed, or \( \frac{\partial \pi(q)}{\partial q} = 0 \))

\[ MR = MC \]

1.3.4 Profit Maximization in the short run

- firm maximizes profits by producing output where \( MR = MC \);
- competitive firm faces a perfectly elastic demand curve, \( MR = P \). Hence, for a perfectly competitive firm, \( P = MC \);
- in short run firms use short run cost curves (SRMC, ATC, AVC) to make profit maximization and shut down decisions;
- firm shuts down if \( P < \text{min AV C} \);
- derive individual firm short run supply curve using \( P = MC \) and \( Q = 0 \) (shut down) for \( P < \text{min AV C} \).
- \( \text{SR market supply curve is horizontal sum of individual firm SR supply curves.} \)
- \( \text{industry profits can be positive or negative in SR.} \)

1.4 Lecture 11- Competition II

1.4.1 Competition in the Long Run

- in LR free entry and exit drives economic profits to 0, i.e. \( P = MC = AC \). Hence, LR industry supply curve is perfectly elastic at \( P = \text{min AC} \) and each firm produces at \( q = \text{arg min AC} \);
- with barriers to entry, problem is as in the SR only firms use their LR cost curves; LR individual supply curve with barriers to entry is LRMC curve above minimum of AC and 0 below.
- \( \text{SR supply less elastic than LR supply with entry barriers, which is less elastic than LR supply with free entry.} \)
- Increasing input prices can lead to an upward sloping LR supply curve even with free entry;
1.4.2 Factor Demand in Competitive Markets

- In SR and in LR, demand for labor will be its marginal revenue product
  
  \[ MRP_L = MR \times MP_L \]

  - where MR is marginal revenue from additional unit of output (MR = p if competitive output market). These will differ in LR and in SR because \( MP_L \) in LR will take into account optimal capital adjustments.

- LR labor demand more elastic than SR

1.5 TO KNOW- Conceptual Understanding

- Explain the difference between the short run and the long run

- Explain why average costs are at a minimum when they cross the marginal cost curve

- Explain/know the condition when a firm will shut down (1) in the short run and (2) in the long run

- Explain when firms will enter/exit in the long run

- Explain why, in theory, long run supply in a perfectly competitive market will be flat at min ATC when there are identical firms

- Know why \( ATC = MC = p \) in the long run for a firm in a perfectly competitive market

- Explain three cases in which long run supply may be upward sloping; for each of these three cases, discuss whether firms earn profits and why

1.6 TO KNOW- Graphical and Math Understanding

- Calculate marginal products given a production function

- Graph isoquants and isocost curves, finding the (L,K) combination that will produce any level of q most cheaply; don’t forget to check for corner solutions

- Know why the firm wants to set \( MPK \times w = MP_L \times r \). Know what this means intuitively and what would happen if this was < or > rather than an equality

- Determine whether a production function exhibits constant, increasing, or decreasing returns to scale

- Calculate and graph various cost curves: ATC, AVC, MC, AFC

- Given input prices and an isoquant, calculate (1) the short run total cost function, (2) long run total cost function, and (3) the long-run expansion path

- Determine whether a production process represented by \( c(q_1; q_2) \) exhibits returns to scope

- Determine based on the production possibilities frontier whether a production process exhibits returns to scope

- In a perfectly competitive market with n firms, calculate residual demand that a single firm faces

- In a perfectly competitive market, given a short run cost curve, find the short run supply curve for a firm
• In a perfectly competitive market, show graphically how aggregate market supply changes as there are more firms

• In a perfectly competitive market in the short-run, given cost curves for firms, demand, and the number of firms, find the equilibrium price, what each firm produces, and the total quantity

• In a perfectly competitive market in the long run, given a cost curve for each firm and demand, determine the equilibrium price, what each firm produces, the total quantity, and the number of firms

• Graph how factor demand and the wage is determined in the labor market a single firm faces

• Given incentives/a compensation scheme, be able to calculate what actions a CEO/manager may take (see problem set 5 for an example)

2 Welfare Economics

2.1 Lecture 12- Competition III

1. Agency Problems

   (a) agency problem when manager of firm does not own full stake in the firm;

   (b) align incentives by using stock options and other payment schemes - can lead to excessive risk taking and short term behavior

2. Stock

   (a) A share of the company

3. Stock Option

   (a) A contract which gives the buyer (the owner) the right, but not the obligation, to buy or sell the stock at a specified strike price on or before a specified date.

   (b) Intuitively, want to exercise a stock option if the price of the stock on the specified date is higher than the strike price. Why? You can buy the stock at the strike price and then sell it at the higher market price. There is no additional cost to not exercising a stock option

4. Expected value

   (a) A random variable X can take the values $x_1, x_2, \ldots, x_k$ and each value occurs with probability $p_1, p_2, \ldots, p_k$. Then the expected value of X is

   $E[X] = x_1 p_1 + x_2 p_2 + \ldots + x_k p_k$

2.2 Lecture 13- Welfare Economics

2.2.1 Consumer Surplus

• The area under the demand curve and above the price since the demand curve represents the marginal willingness to pay for a good

2.2.2 Producer Surplus

• The area above the supply curve and below the price since the supply curve represents the marginal cost of producing the good
2.2.3 Total Welfare
- The addition of consumer and producer surplus
- It is maximized under perfect competition—when demand = supply

2.2.4 Deadweight Loss
- The loss in welfare that is a result of moving away from the perfectly competitive equilibrium
- Can be caused by monopolies, government taxation, etc

2.3 TO KNOW- Conceptual Understanding
- Explain how consumer surplus depends on the elasticity of the demand curve
- Explain what deadweight loss is intuitively
- Explain why competition maximizes total surplus

2.4 TO KNOW- Graphical and Math Understanding
- Know how to calculate consumer surplus, producer surplus, and deadweight loss from various government policies (quantity restriction, price ceiling, price floor, tax, etc.)

3 Monopoly
3.1 Lecture 14 - Monopoly I
3.1.1 Monopoly Profit Maximization –
- Total Revenue is
  \[ TR = P(Q) \cdot Q; \]
- Average Revenue for a firm is given by demand curve
  \[ AR = P(Q); \]
- Marginal Revenue is additional revenue from selling one more unit,
  \[ MR = \frac{\partial TR}{\partial Q} \]
- Perfectly Competitive firm faces a perfectly elastic demand curve, \( P(Q) = P \) and hence, \( MR = P = AR \).
- Monopoly faces downward sloping demand curve and hence
  \[ MR = \frac{\partial TR}{\partial Q} = \frac{\partial P(Q) \cdot Q}{\partial Q} = P(Q) + Q \frac{\partial P}{\partial Q} \]
  Notice that
  \[ MR = P(Q) + Q \frac{\partial P}{\partial Q} < P(Q) \]
  since \( \frac{\partial P}{\partial Q} < 0 \).
• Monopolist has to decrease price on all units sold in order to sell one additional unit. This is not the case with a perfectly competitive firm, which cannot influence the price at which it sells.

  - MR curve for monopolist is below AR curve (the demand curve)

• A monopoly never produces at the inelastic part of the demand curve

\[ MR = \frac{\partial TR}{\partial Q} = \frac{\partial (P(Q)\cdot Q)}{\partial Q} = P(Q) + QP' = P(Q)(1 + \frac{Q}{P(Q)}) \]

For \( |\epsilon_D| < 1 \), \( MR < 0 \)

• Profit maximization

  - \( \Rightarrow MR = MC \).

\[ MR = P(1 + \frac{1}{\epsilon_D}) = MC \]

Markup\( \rightarrow \frac{P-MC}{P} = -\frac{1}{\epsilon_D} \)

• Hence mark-up, measure of monopoly power, depends on the elasticity of demand

• Shut down decision is like that of a competitive firm.

### 3.1.2 Welfare Effects of Monopoly

• Because \( MR < AR \), monopolist would supply less than the socially optimal (welfare maximizing) level of output, which leads to a deadweight loss

### 3.2 Lecture 15 - Monopoly II

#### 3.2.1 I. Price Discrimination –

• above analysis is for a uniform pricing monopoly

• the monopolist sets the same price for every unit sold or for every consumer type.

• monopolist can price discriminate - set different prices for different units, charge different uniform prices for different consumer groups, use two part tariffs, etc.

• perfect price discrimination/1st degree price discrimination

  - monopolist charges each consumer their willingness to pay for the good, and hence extracts all the consumer surplus. MR curve is now the AR curve, i.e. the demand curve.

  - Set output where new MR curve equals MC, i.e. where demand intersects MC.

  - hence, a perfectly price discriminating monopolist produces the socially optimal output level.

#### 3.2.2 How do Monopolies Arise?

1. cost advantages - natural monopoly, for any output produce at lower AC than any other firm can (AC is declining);

2. barriers to entry - fixed costs, patents;
3.2.3 Regulating Monopolies

- Government regulation of monopoly, through a price ceiling can improve welfare.
- Setting a price ceiling at the competitive price leads to zero DWL.
- Effect of a unit tax on the price of good - price of good can increase by less than 1 for 1 with the tax or by more
- difference with perfectly competitive market.

3.2.4 IV. Contestable Markets –

- threat of entry “disciplines” monopolist and they charge a price close to the perfectly competitive price.

3.3 TO KNOW- Conceptual Understanding

- Explain why marginal revenue is less than average revenue for a monopolist but not for a competitive firm
- Know why both a monopolist and perfectly competitive firm want to set MR = MC
- Explain why a monopolist’s market power depends on the elasticity of demand
- Explain why there is deadweight loss (DWL) when a monopolist cannot price discriminate
- Explain why there is no deadweight loss (DWL) when a monopolist can price discriminate
- Know reasons monopoly may rise
- Discuss the pros and cons of patents

3.4 TO KNOW- Graphical and Math Understanding

- Given a cost function and a demand curve, solve for the price and quantity in a market with a monopolist; be sure to check whether the monopolist will want to shut down
- Derive an equation relating the monopolist markup to the elasticity of demand
- Graphically, identify the producer surplus, consumer surplus, and DWL of monopoly in the uniform price case
- Graphically, identify producer surplus, consumer surplus, and DWL when a monopolist can perfectly price discriminate
- Graphically show the welfare impact of patents
- Graphically show the welfare effects of government regulation of monopolies (a) Three cases:
  - mandated price above p , mandated price below p; mandated price at p
4 Other Market Structures (Chapter 13)

4.1 Lecture 16 - Other Market Structures

4.1.1 Oligopoly

- small number of firms that interact strategically (not price takers but have to take into account other firms’ decisions when making their own decisions);
- duopoly - market with two firms;

4.1.2 Game Theory –

- study of the outcome of strategic interactions;
- player’s objective is to maximize payoffs given actions of others;
- non-cooperative games - players cannot enforce mutually beneficial strategies;
- strategies - possible actions that players choose from to maximized payoffs;
- dominant strategy - strategy that maximizes a player’s payoff no matter what the other player does;
- Nash equilibrium - each player is doing the best it can (maximized payoff) given the actions of its opponents;

4.1.3 Cournot Model of Noncooperative Equilibrium

- Cournot duopoly - two firms compete by setting output levels simultaneously. Each firm treats the output of its competitor as fixed;
- Reaction curve - relationship between firm’s profit maximizing output and output it thinks its competitor will produce;
- Cournot equilibrium - Nash equilibrium of Cournot duopoly game. Output levels for which reaction curves intersect.
- Cournot Math : All firms set quantities at the same time

1. Calculate residual demand for a given firm (in other words, the demand for a firm’s product subtracting out other firms’ output decisions)

2. Create a total revenues function

3. From the total revenues function, derive marginal revenues

4. Solve its profit maximization problem (MR=MC). This will give you a firm’s best response function to other firms’ output decisions.

5. Solution is a set of quantities (one for each firm) that solves the system of equations in 4.
4.2 Lecture 17 - Oligopoly continued I

- Cooperative Equilibrium - Cartels
  - Firms can form a cartel and behave like a single monopolist, maximizing total industry profits.
  - Cartels are unusual because they are fundamentally unstable (incentive to “cheat” and raise own production) and because they are illegal (antitrust laws).

- Comparing Equilibria –
  - In terms of welfare, usually Perfect Competition > Oligopoly > Monopoly
  - Quantity as an indicator of social welfare
  - DWL in welfare analysis comes from trades that aren’t made

- Many Firms
  - In Cournot, as number of firms → ∞, Cournot equilibrium approaches competitive equilibrium
  - As number of firms → 1, approaches monopoly
  - Markup over competitive price depends on number of firms and elasticity of demand:
    \[
    \frac{P - MC}{P} = -\frac{1}{n \times \epsilon_D}
    \]

- Price Competition
  - Bertrand: firms set prices (instead of quantities) at the same time
  - Two firms may be enough to remove market power (i.e. restore competitive outcome) if products are identical
  - Recall proof from class that identical Bertrand duopolists drive price down to marginal cost. In other words, firms who compete a la Betrand, will set their prices at marginal cost.

4.3 TO KNOW- Conceptual Understanding

- Explain the “prisoner’s dilemma”
- Understand why cooperation can be sustained in a infinitely repeated game but not in a game with finite periods
- Explain why cartels are unstable
- Compare welfare from different forms of competition (monopoly, oligopoly, perfect competition)
- Know the difference between quantity (Cournot) and price (Bertrand) competition

4.4 TO KNOW- Graphical and Math Understanding

- Find the Nash equilibrium of a game, given a payoff matrix
- Solve for quantities and prices when two firms compete in Cournot equilibrium
- Solve for a cartel equilibrium with n firms
- Solve for price and quantity when firms compete in a model of Betrand price competition
14.01 Principles of Microeconomics
Fall 2018

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