Recitation 5 Notes
14.01SC Principles of Microeconomics

I. Production Functions: $Q = f(L,K)$
   a.) Cobb-Douglas production function
      $$Q = L^{\frac{1}{2}} K^{\frac{1}{2}}$$
      $$MPL = \frac{\Delta Q}{\Delta L} = \frac{\partial Q}{\partial L} = \frac{1}{2}(K/L)^{\frac{1}{2}}$$
      $$MPK = \frac{\partial Q}{\partial K} = \frac{1}{2}K^{\frac{1}{2}}L^{\frac{1}{2}} = \frac{1}{2}(L/K)^{\frac{1}{2}}$$
      Marginal rate of technical substitution = $-\frac{dK}{dL}$
      $$(\text{MRTS}) = \frac{MPL}{MPK}$$
      $$= \frac{(.5*(K/L)^{.5})}{(.5*(L/K)^{.5})}$$
      $$= K/L$$
      Returns to scale are constant.
   b.) Leontief
      $$Q = \min(aL, bK)$$
      $$aL = bK$$

II. Costs (forward-looking)
   a.) Short Run Costs
      K is fixed
      Fixed cost (FC) = $r * k$
      Variable cost (VC) = $w * L$
      Marginal cost (MC) = $\Delta C/\Delta Q = dC/dQ$
      Average fixed cost (AFC) = FC/Q
      Average variable cost (AVC) = VC/Q
      Average total cost (ATC) = AFC + AVC
      Short run cost (SRC) = $r * K + w * L$
   b.) Long Run Costs
      MRTS = $w/r$
      $$\frac{MPL}{MPK} = \frac{w}{r} \Rightarrow MPL/w = MPK/r$$
      Long run costs: $LRC(Q) = r*K(Q) + w*L(Q)$
      Increasing returns to scale if Q increases and AC decreases
      Decreasing returns to scale if Q decreases and AC increases