14.02 Principles of Macroeconomics
Problem Set 6
Fall 2004

Part I. True/False/Uncertain
Justify your answer with a short argument.

1. In an economy with technological progress, the saving rate is irrelevant in the long-run.

2. The rate of technological progress declined in the mid-1970s due to the rise of the service sector.

3. There is much theoretical and empirical support for the idea that faster productivity growth leads to higher unemployment.

4. Increased globalization and technological progress can both be blamed, at least to some extent, for the increase in wage inequality in the United States.

5. The Fisher hypothesis states that, in the short-run, the nominal interest rate increases one for one with inflation, while the real interest rate remains unchanged.

Part II. The Solow Model Revisited

The Republic of Solowakia has the following production function:
\[ Y = F(K, N) = AK^\alpha N^{1-\alpha}, \text{ where } \alpha < 1. \]

Let \( g_A \) be the growth rate of \( A \), \( g_N \) be the growth rate of \( N \), and \( \delta \) be the rate of depreciation in this economy.

1. Interpret the parameter \( A \). Empirically, what has happened to \( A \) over time?

2. Verify that the above production function has the property of constant returns to scale.

3. Verify that the above production function is concave in capital (which means that the second derivative is negative). What does that mean in economics terms?

4. What is effective labor in this economy?

5. At what rate do output and capital grow in Solowakia in the long run? (Hint: use \( g_A \) for the transformed \( g_A \).)
6. Rewrite the production function in terms of only capital per effective worker.

7. Draw the diagram that plots required investment and investment for this economy. Label the steady-state level of capital per effective worker $k_1^*$. Label the steady state as point A on the diagram.

8. Suppose the rate of population growths falls. Use a diagram to analyze what happens in Solowakia. Label the new steady state capital per effective worker $k_2^*$. Label the new steady state as point B.

9. Starting at the steady state in part 7 (point A), suppose the saving rate increases and the depreciation rate decreases at the same time. Use a diagram to analyze what happens in Solowakia. Label the new steady state capital per effective worker $k_3^*$. Label the new steady state as point C.

10. Starting at the steady state in part 7 (point A), suppose a neighboring country gives Solowakia a gift of new capital equipment. Use a diagram to analyze what happens in the long run. Label the new steady state capital per effective worker $k_4^*$. Label the new steady state as point D.

Part III. Expected Present Discounted Values

1. Suppose that the interest rate is 5% today and is expected to stay at 5% for the next three years. Calculate the present discounted value of a security that pays $10,500 in one year, $11,025 in two years, $23,152.5 in three years, and nothing thereafter.

2. Suppose the security in part 1 sells for $38,342.08 today. Would its discount rate need to be smaller or higher than 5%? Answer without computing it first. Then calculate the discount rate. Assume again that the interest rate is expected to remain the same for the next three years.

3. What is the constant payment required to achieve the same present discounted value over three years as in part 2 (i.e. $38,342.08), at the same constant interest rate as you found in part 2?

4. Suppose that the security pays the annual payment you calculated in part 3 forever at the constant interest rate from part 2. Calculate the present discounted value of this security.