

14.02 Principles of Macroeconomics

Spring 2014

Problem Set 1

Due: February 21, 2014

Question 1 (20 points)

Consider the balance sheet of an American bank before the crisis. On the liabilities side, the bank has a total capital of 10 billion dollars and deposits worth of 150 billion dollars. On the assets side, 10% of the assets are subprime mortgages. The rest of assets are considered safe and will not go through changes in value.

1. What is the leverage of this bank? (6 points)

Solution:

$$\lambda = \frac{10 + 150}{10} = 16.$$

2. Suppose that after the outbreak of the crisis, the value of the subprime loans falls by a half. Is the bank still solvent? What is its new leverage? (6 points)

Solution:

The total amount of subprime mortgages that the bank has is $160 \times 10\% = 16$ billion dollars before the crisis. After the crisis, the value of them fell in half and become 8 billion dollars. The bank takes a hit of 8 billion in its capital, but it remains solvent. Its remaining capital is $10 - 8 = 2$ billion dollars.

$$\lambda' = \frac{2 + 150}{2} = 76.$$

3. According to the Basel III rule (the international rule that determines banks' minimum required capital, try to google it out if you haven't heard about it before), banks are supposed to have a leverage ratio of at most $\frac{100}{3}$. Does this bank need to recapitalize? If so, by how much? (8 points)

Solution:

Since $76 > \frac{100}{3}$, the bank needs to recapitalize. Let x be the amount of recapitalization. The bank's leverage after recapitalization would be

$$\lambda'' = \frac{2 + x + 150}{2 + x}.$$

$$\lambda'' \leq \frac{100}{3} \text{ if and only if } \frac{2+x+150}{2+x} \leq \frac{100}{3}, \text{ or } x \geq \frac{256}{97}.$$

Question 2 (30 points)

Assume a bank has a certain amount of debt, D , that is 5 times as much as its own capital. Assume that a third of the bank's money is invested in some risky investment with a random rate of return θ (θ dollars back for 1 dollar invested). Assume that θ follows a uniform distribution with a support ranging between 0 and 2.

1. Draw the bank's balance sheets today and tomorrow, assuming it will be solvent. (10 points)

Solution:

The bank's balance sheet today looks like

Assets	Liabilities
Risky Assets: $\frac{2}{5}D$	Debt: D
Safe Assets: $\frac{4}{5}D$	Capital: $\frac{1}{5}D$

The bank's balance sheet tomorrow will look like

Assets	Liabilities
Risky Assets: $\frac{2}{5}\theta D$	Debt: D
Safe Assets: $\frac{4}{5}D$	Capital: $\frac{1}{5}D + \frac{2}{5}(\theta - 1)D$

2. Determine the critical value of θ below which the bank is not solvent. What is the probability that this happens? (10 points)

Solution:

The bank will be solvent tomorrow if and only if its capital stays positive:

$$\frac{1}{5}D + \frac{2}{5}(\theta - 1)D \geq 0,$$

which gets simplified to $\theta \geq 1/2$. The odds that this would happen is 75% (or alternatively, the odds that the bank becomes insolvent is 25%).

3. Assume that the bank wants to stay solvent in 90% of cases. What can it do? (Hint: you should think about both reducing its exposure to risks or increase its capital.) (10 points)

Solution:

The bank can either reduce its exposure in risky assets or increase its capital.

Suppose first that it reduces its shares of investment in risky assets to a fraction of α of its total assets. The bank's capital tomorrow will be

$$K' = \frac{1}{5}D + \frac{6}{5}\alpha(\theta - 1)D.$$

Making sure it is solvent in 90% of cases means K' has to be non-negative for any value of $\theta \geq 0.2$. So

$$\frac{1}{5}D + \frac{6}{5}\alpha(0.2 - 1)D \geq 0,$$

which gets simplified to $\alpha \leq \frac{5}{24}$.

Suppose now that the bank tries to increase its capital. Let ΔK be the amount of capital increase that the bank takes. The bank's capital tomorrow will be

$$K' = \frac{1}{5}D + \Delta K + \frac{2}{5}(\theta - 1)D.$$

Making sure it is solvent in 90% of cases means K' has to be non-negative for any value of $\theta \geq 0.2$. So

$$\frac{1}{5}D + \Delta K + \frac{2}{5}(0.2 - 1)D \geq 0,$$

which gets simplified to $\Delta K \geq 0.12D$.

To summarize, the bank can either reduce its exposure in risky assets to $\frac{5}{24}$ of total assets (or $\frac{1}{4}D$) or increase its capital by $0.12D$ (A 60% increase in capital).

Question 3 (50 points)

Consider the standard version of the Holmstrom Tirole model we discussed in class.

1. Consider first the case of direct investment (i.e. with no banks). Prove that there is a minimum amount of net worth, \bar{A} , that entrepreneurs need to have to start the projects. (10 points)

Solution:

Entrepreneurs' inventive compatibility (I.C.) constraint is

$$R_E \geq \frac{B}{\Delta p}.$$

Small investors' individual rationality (I.R.) constraint (which makes sure they want to take part in the project investment) is

$$p_H R_S \geq \gamma(I - A).$$

Since $R = R_E + R_S$, these imply

$$R \geq \frac{B}{\Delta p} + \frac{\gamma}{p_H}(I - A),$$

which gets simplified to

$$A \geq \bar{A} = I - \frac{p_H}{\gamma} \left(R - \frac{B}{\Delta p} \right).$$

This proves the necessity of this condition. To see this is also sufficient, whenever this holds, define

$$R_S = \gamma(I - \bar{A}).$$

One could check easily that the I.C. and the I.R. conditions are satisfied, which proves the sufficiency.

2. Now suppose that no entrepreneurs in this economy have sufficient net worth so no projects are started in equilibrium. If you are a social planner, is

there a way to redistribute initial wealth (only before the projects get started) in this economy to make a Pareto improvement (a Pareto improvement is a situation in which all agents in this economy are better-off)? What happens if instead of focusing on Pareto improvements, you only care about the GDP (or total investment)? (12 points)

Solution:

A social planner **cannot** make a Pareto improvement with redistribution of initial wealth alone. The reason is the following. The only way to get a Pareto improvement is to make it possible to start some projects that are not started in equilibrium. The way to do so is either by redistributing wealth from some entrepreneurs or small investors to other entrepreneurs (who have less than \bar{A}). The situations are the same regardless of whose money is taken from, so in what follows, assume the social planner takes money from small investors.

The only way this redistribution might be Pareto-improving is by letting the small investors and the entrepreneur willingly sign a contract between them to make sure that the former is rewarded more than the amount of wealth that he is taken away of, plus the additional investment into the project if there is any. The problem is that this contract will not be incentive compatible - if it were, both parties would be willing to do it even without social planner's intervention.

To make this argument clearer, consider the following numerical example. Entrepreneurs in this economy have, say, 10 (thousand) dollars. With $\bar{A} = 12$ and $I = 15$, this is not enough for them to start projects. The aforementioned way of trying to make a Pareto-improvement is to redistribute 2 from small investors to the entrepreneurs. By doing so, the entrepreneurs' wealth reaches \bar{A} , thus they are able to get 3 more from small investors to start the projects. However, to make this arrangement Pareto-improving, it has to be the case that small investors will be rewarded (in expectation) no less than $2+3 = 5$. Yet this is not possible. Suppose by contradiction that there is a contract between them that makes this happen. Consider a new contract that is exactly the same as this one, except that the initial investment money is increased by 2 (the exact amount of redistribution). One can easily verify that this new contract should be able to get the projects started without the redistribution. The fact that this contradicts the definition of \bar{A} proves the impossibility of a Pareto-improvement.

For those of you who know the first welfare theorem, another way of saying the same thing is that the original equilibrium is (constraint-) efficient. So a social planner cannot improve it.

If instead of focusing on Pareto improvements, the social planner care about the GDP or total investment, he can clearly increase that by both above-mentioned redistributions, as there will be more projects started in the equilibrium.

3. Now let's put banks back into the story. Recall that we denote by β the rate of return that a bank can obtain if it invests elsewhere. Show that there is a minimum amount of capital, I_B , that banks have to contribute to the projects for there to be active monitoring. (10 points)

Solution:

Banks' incentive compatibility constraint is the following:

$$R_B \geq \frac{c}{\Delta p},$$

where c is the monitoring cost.

Banks' individual rationality constraint (making sure they earn weakly more in the investment) is

$$\beta I_B = p_H R_B - c,$$

which implies

$$I_B \geq \frac{p_L}{\beta \Delta p} c,$$

which is the minimum amount of capital that banks have to contribute to the projects for there to be active monitoring.

4. Assume that $b + c < B$. Compare a firm's pledgeable income with or without banks. Show that in this case, it is possible for banks to possess zero capital yet they still take part in monitoring. What happens if $b + c > B$? (13 points)

Solution:

Assume that $b + c < B$. A firm's pledgeable income is $R - B/\Delta p$ without the banks, which is smaller than $R - (b + c)/\Delta p$, its pledgeable income with banks. This means that monitoring would allow a firm to raise more uninformed capital from small investors than without monitoring. As a result, more projects will be started and banks' in this case will earn a positive profit, R_B , despite the fact that they invest nothing. For those of you who think this might not be consistent with the previous question, the way it works out (mathematically) is to have $I_B = 0$ and $\beta = +\infty$.

If $b + c > B$, however, monitoring makes firms' pledgeable income smaller therefore if banks invest nothing on their own, monitoring would not be an interesting option for entrepreneurs as this would make it harder to attract uninformed capital from small investors. As a result, there is no monitoring in this situation.

5. What happens for entrepreneurs if $A < \bar{A}$ yet $A + I_B > I$? (5 points)

Solution:

These entrepreneurs cannot start projects on their own, yet they would have more than the required amount of investment with informed capital from banks alone. Therefore, these firms will only demand informed capital and invest their excess fund in the market.

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