14.02 Exam 1 Solution

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March 19, 2012

1 True/False Questions: (5 points each)

Note: The answers should be justified by short sentences.

1. The increase in house prices from 2000 to 2006 contributed to the decline in the foreclosure rate of homes during those years.
   T, because even if households did not have enough money, many of them were able to borrow against the increased value of their home and pay back their debt. Also the strategic default when home prices are increasing cannot exist.

2. A bank has $10 worth of capital and its leverage is 5. If the bank is paying zero interest rate on its debt, a 2% increase in asset prices results in a 10% return on the capital of the bank.
   T, total asset of the bank: $5 \times 10 = 50$, plus 2% return: $51 - \text{minus debt}(=40) \rightarrow 11 \rightarrow 10\% \text{ return on capital.}$

3. The higher leverage of European banks compared to US banks means that the probability of European banks going bankrupt is higher than the same probability for US banks.
   F, probability of bankruptcy is function of both the leverage of the bank and the bank exposure to risky assets. The main reason that European banks had higher leverage ratio was that they were holding more government bonds—which was perceived as safe assets—and they had less exposure to risky assets.

4. In the Holmstrom-Tirole model, an increase in the return to safe assets ($\gamma$) decreases the number of projects that can be financed directly—meaning by small investors only.
   T, This is because with higher $\gamma$ investors are asking for higher returns and therefore there are less resources left for the entrepreneurs to be incentivized. In terms of the model, this will result to a higher $A(\gamma)$. 


5. Provided there is enough liquidity in the economy, the market will always allocate it to the firms that face an unexpected liquidity shortage.

F. Ex-post banks may not have the right incentive to provide the liquidity to those who needs it.

6. A 10% fall in investment has the same impact on GDP as a 10% fall in consumption.

F. Simply because in the data consumption is much larger than investment.

7. Suppose that the government introduces a new spending program that is fully paid for by an increase in taxes. Such a program will have no effect on investment.

T. As you saw in the lectures such a program will increase $Y$ with the same amount as change in $G$. $(dY = dG)$. This means the consumption will not change since $C = c_0 + c_1(Y − T)$ (or $dC = c_1(dY − dT)$) and therefore $I = Y − C − G$ is not changing.

8. Higher nominal interest rates increase the demand for cash.

F. higher nominal interest rates increases the shadow cost of holding money and therefore increase the demand for bonds and reduces the demand for money.

2 Short Long Question: IS-LM Question (30 points)

Consider an economy described by the following equations:

$$C = a + b(Y − T)$$
$$I = cY − d_i + e$$
$$\frac{M^d}{P} = fY − gi$$
$$G = G$$
$$\frac{M^s}{P} = M$$
$$P = 1$$

1. Derive an expression for the IS-curve and explain what does this equation represent. (5 points)

$$Y = C + I + G$$
$$= a + b(Y − T) + cY − d_i + e + G$$

$$Y = \frac{a − bT + e + G}{1 − b − c} − \frac{di}{1 − b − c}$$

Equilibrium in the goods market
2. Derive an expression for the LM-curve and explain what does this equation represent. (5 points)

\[ fY - gi = M \]

\[ Y = \frac{g}{f} i + \frac{M}{f} \text{ or } i = \frac{fY - M}{g} \]

Equilibrium in the financial market.

3. Compute the level of private saving when both the goods and the financial market are in equilibrium. (10 points)

They should solve for the two equations-two unknowns

\[ Y = \frac{a - bT + e + \bar{G} + \frac{d}{g} \bar{M}}{1 - b - c + \frac{df}{g}} \]

and then we have \( S = Y - C \) and therefore

\[ S = (1 - b)Y - a + bT \]

\[ = \frac{1 - b}{1 - b - c + \frac{df}{g}} (a - bT + e + \bar{G} + \frac{d}{g} \bar{M}) - a + bT \]

4. Keeping government revenue \( (T) \) constant, what is the impact of an increase in government spending \( (G) \) on total output and private saving? Show the impact on output using an IS-LM graph. (5 points)

\[ \Delta Y = \frac{1}{1 - b - c + \frac{df}{g}} \Delta G > 0 \]

\[ \Delta S = \frac{1 - b}{1 - b - c + \frac{df}{g}} \Delta G > 0 \]

in IS-LM curve, IS is shifting to the right \( \rightarrow \) higher \( Y \) and higher \( i \)

5. Just by using the IS-LM graph show what is the impact of an increase in the supply of money \( (\bar{M}) \) on output and investment? (5 points)

in IS-LM, LM is shifting to the right \( \rightarrow \) higher \( Y \) and lower \( i \).

then for the investment we have

\[ \Delta I = c\Delta Y - d\Delta i > 0 \]
3 Long Long Question: A Modification of the Diamond-Dybvig model (30 points)

Consider the standard version of the Diamond-Dybvig model discussed in class.

A bank has access to two investment technologies. There is a short term technology which yields 1 in period $t = 1$ for every unit invested in period $t = 0$. There is also a long term technology which yields $R > 1$ in period $t = 2$ for every unit invested. The bank can choose to liquidate its long term investment in $t = 1$, in which case it gets $L < 1$ units for each unit invested at $t = 0$.

Each consumer has 1 unit of endowment and the total size of the population is normalized to 1. In period $t = 0$ consumers do not know if they will be impatient or patient. In period $t = 1$, a fraction $1 - \pi$ of consumers realise that they are impatient, which means they want to consume their endowment immediately. A fraction $\pi$ of consumers find out that they are patient, which means that they only enjoy consuming at $t = 2$.

The bank offers a contract which gives right to withdrawals of size $(c_1^+, c_2^*)$ in period $t = 1$ and $t = 2$ respectively to consumers who in $t = 0$ deposit their endowment at the bank.

1. Assume $c_1^+ > 1$. What is the maximum amount $c_2^*$ that a bank could promise to patient consumers under the assumption that only impatient consumers will withdraw $c_1^+$ at $t = 1$? (5 points)

\[
(1 - \pi)c_1^+ + \frac{\pi c_2^*}{R} = 1
\]

This is because if the bank wants to provide $c_2^*$ for the patient consumers, the bank should invest $\frac{\pi c_2^*}{R}$ in the long-term projects. But the total resources available for the bank is $1 - (1 - \pi)c_1^+$.

2. How much should the bank invest in each technology to fulfill its promise of $(c_1^+, c_2^*)$? (5 points)

\[
\frac{\pi c_2^*}{R}
\]

Now we change our assumptions:

We assume that instead of the bank there is a firm which has access to exactly the same two technologies as the bank. The only difference is that instead of the deposit contract, for each dollar invested in the firm, the firm offers a share. A "share" is the right to receive a payment $D_1$ at $t = 1$ and $D_2$ at $t = 2$ independent of whether who owns the share is patient or impatient. In $t = 1$, consumers can sell or buy the shares of the firm, only after receiving $D_1$ and finding out whether they are impatient or patient.

The sequence of events is as follows:

In $t = 0$ people give their one unit of endowment to the firm.
In $t = 1$ the firm pays $D_1$. After $D_1$ has been paid and consumers find out whether they are patient or impatient, consumers can sell or buy the shares of the firm.

In $t = 2$ the firm pays $D_2 = R(1 - D_1)$ since we are assuming the firm is making zero profit.

3. Let us first assume the price of the shares is such that patient consumers can buy all of the shares of impatient consumers by giving them their $D_1$ dividends. Show that if $D_1 = (1 - \pi)c_1^*$ then impatient consumers consume $c_1^*$ and patient consumers consume $c_2^*$. (5 points)

If patient consumers are spending all of their $t = 1$ dividends to pay for the shares, this means impatient consumers in total receive $D_1$ and therefore impatient consumers receive $\frac{D_1}{(1 - \pi)}$ per person. Therefore if $c_1^* = \frac{D_1}{(1 - \pi)}$, each impatient consumer can consume $c_1^*$.

4. Find the equilibrium price of shares in $t = 1$ (call it $P_s$) that allows patient consumers to buy all the shares from impatient consumers using their own dividends $D_1$. (Hint: The total expenditure by patient consumers must equal the value of all shares sold by impatient consumers) (10 points)

The total spending on the shares by the patient consumers is $\pi D_1$ whereas the total value of the sold shares is $(1 - \pi)P_s$. Therefore in the equilibrium we have:

$$ (1 - \pi)P_s = \pi D_1 $$

$$ P_s = \frac{\pi}{1 - \pi} D_1 $$

5. Is it possible to have a bank-run (i.e. the firm is liquidated although it is solvent) in this case? Explain the intuition. (5 points)

No, because even if the patient consumers decide to not buy the shares at $t = 1$, they cannot liquidate the firm. This is because the firm has already paid its promise $D_1$ and does not have any other liability for $t = 1$. Moreover if one impatient investor decides to not buy any share, this will just reduce the equilibrium price which in turn increases the demand of other patient consumers for buying the shares. Therefore there cannot be any self-fulfilling run in this case.