Outline of the Course

Part 1: Origins of the 2007-13 economic crisis: why did a failure of the financial system (of banks in particular) produce the largest recession since the Great Depression of the 1930s?

- The financial system (banks) is the core of the economy. We cannot understand macroeconomics if we do not understand how shocks and policies affect the economy through their effects on the financial system, banks in particular
  - Why do banks exist? Why are they fragile institutions?
  - Four small models
    - Leverage, Moral Hazard, Maturity Mismatch
    - Borrowing constraints & amplification of economic shocks
Part 2: The policy response to the financial crisis

- Background models ("traditional" macroeconomics)
- The Fed’s response: interest rates and "quantitative easing"
- Fiscal policy: President Obama’s "Stimulus Program"
- Did these policies work?
- Legacy of the crisis: Debt and the Eurocrisis
Macroeconomics and the Financial System

- Understanding banks
  - balance sheets
  - leverage
  - non-linearities
The balance sheets of banks and other financial firms are central to understanding how a financial system works and why in 2007-08 it blew up.

Before studying the financial system we thus need to understand what a balance sheet is. We do it in 3 steps:

- definition
- balance sheet of a household
- balance sheet of a bank
Balance sheets explained to your younger brother

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saving for a rainy day</td>
<td>Other people’s money</td>
</tr>
<tr>
<td>Money working for you</td>
<td>Own skin in the game</td>
</tr>
</tbody>
</table>

- Assume you wished to set up a company and to start you need 100K$. You have 10K of your own (your skin in the game) and borrow 90K from a bank. You then spend 95k to start the company (this is the money working for you) and leave 5K in the bank as a buffer in case something (small) goes wrong.

- You can loose Your skin in the game. If you loose more, i.e. Other people’s money, you are bankrupt. Your skin in the game is the buffer against possible losses: the smaller the buffer, the smaller the losses you can withstand without going bankrupt.

- Accountants call
  - Other people’s money: Debt
  - Your Skin in the Game: Equity
  - Saving for a rainy day: Reserves
  - Leverage = \( \frac{\text{Assets}}{\text{Your Skin in the Game}} = \frac{\text{Assets}}{\text{Equity}} \)
Example: the balance sheet of a household

- Consider a household which bought a house financed by a mortgage.
- How large is the mortgage as a fraction of the value of the house obviously makes a big difference.
- Also the type of mortgage makes a difference: is the interest rate fixed, or does it float with market rates? At current interest rates and at current house prices the household may look perfectly able to make the mortgage payments, but what if house prices fall, or interest rates rise?
- To understand how risky is the position of this household we need to know its balance sheet, i.e. the value of the house and the size and conditions of the mortgage.
## Balance sheet of a household (thousand US $)

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>House</td>
<td>Mortgage</td>
</tr>
<tr>
<td>1.000</td>
<td>900</td>
</tr>
<tr>
<td>Stocks</td>
<td>Equity</td>
</tr>
<tr>
<td>50</td>
<td>160</td>
</tr>
<tr>
<td>Bank deposits</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

- this family has purchased a house with a downpayment of 100 and a mortgage worth 900. Its net worth (its *Equity*) is 160: 100 (equity in the house) + 60 (cash and stocks)
- its leverage (the ratio of Assets to the family’s net worth) is 1060/160 = 6.625
Assume house prices fall 30% and the value of the house falls to 700. The family is broke: it’s net worth has become negative: $60 + (100 − 300) = −140$ (because the 100 of equity in the house is less than the fall in the value of the house).

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>House</td>
<td>Mortgage</td>
</tr>
<tr>
<td>700</td>
<td>900</td>
</tr>
<tr>
<td>Stocks</td>
<td>Net worth</td>
</tr>
<tr>
<td>50</td>
<td>−140</td>
</tr>
<tr>
<td>Bank deposits</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

If the interest rate on the mortgage remains unchanged, he family is still able to make its monthly mortgage payment: just looking at flows (monthly income and monthly mortgage payments) we would not have guessed the family could be in trouble.

The problem is that this family had too much debt. What would have happened if its leverage had been 2 instead of 6.625?

But what does "being broke" mean in practice?
Assume the mortgage is a *non-recourse loan*, *i.e.* if the borrower is delinquent the bank has only the right to re-possess the house. Now the bank is broke (its equity, 140 is not sufficient to absorb the loss, 200)

<table>
<thead>
<tr>
<th>household</th>
<th>bank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assets</td>
<td>Liabilities</td>
</tr>
<tr>
<td>House</td>
<td>1000</td>
</tr>
<tr>
<td>Stocks</td>
<td>50</td>
</tr>
<tr>
<td>Bank deposits</td>
<td>10</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>household</th>
<th>bank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assets</td>
<td>Liabilities</td>
</tr>
<tr>
<td>House</td>
<td>0</td>
</tr>
<tr>
<td>Stocks</td>
<td>50</td>
</tr>
<tr>
<td>Bank deposits</td>
<td>10</td>
</tr>
</tbody>
</table>
Assume instead that the mortgage contract gives the bank the right to reclaim not only the house, but any other households’ assets. Now the bank survives: its equity is zero but not negative.

<table>
<thead>
<tr>
<th>household</th>
<th>bank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assets</td>
<td>Liabilities</td>
</tr>
<tr>
<td>House</td>
<td>1000</td>
</tr>
<tr>
<td>Stocks</td>
<td>50</td>
</tr>
<tr>
<td>Bank deposits</td>
<td>10</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>household</th>
<th>bank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assets</td>
<td>Liabilities</td>
</tr>
<tr>
<td>House</td>
<td>0</td>
</tr>
<tr>
<td>Stocks</td>
<td>0</td>
</tr>
<tr>
<td>Bank deposits</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>household</th>
<th>bank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assets</td>
<td>Liabilities</td>
</tr>
<tr>
<td>House</td>
<td>0</td>
</tr>
<tr>
<td>Stocks</td>
<td>0</td>
</tr>
<tr>
<td>Bank deposits</td>
<td>0</td>
</tr>
</tbody>
</table>
The Balance Sheet of a Bank

- Remember what is Leverage

\[ \text{Leverage} = \frac{\text{Assets}}{\text{Your Skin in the Game}} = \frac{\text{Assets}}{\text{Equity}} \]
Leverage and Fragility

- The balance sheet of banks is crucial to understand
  - their role in transferring savings from households to firms
  - why they are fragile institutions
- remember that the reason banks hold equity is to absorb possible losses on the assets they own
- how much equity should a bank have is the central question in the current debate about reforming the financial system and thus avoid another financial disaster
- Thus there is a tradeoff
  - the more equity a bank holds, the larger its buffer, the stronger the bank is, i.e. the less likely it goes bankrupt
  - the more equity a bank has, the lower its profitability: this explains why raising equity is difficult
Leverage

- Assume a bank has an amount of debt (e.g. deposits) $D$ and an amount of equity ($\bar{K}$) also called capital. Its liabilities are $L = D + \bar{K}$, equal to its total assets.

- The bank holds two types of assets:
  - loans and other investments (what we called Money working for you)
  - reserves (what we called Savings for a rainy day)

- Let $\alpha$ be the fraction of total assets invested, and $(1 - \alpha)$ the fraction kept as reserves.

- Investment is risky: for each dollar invested today you get $p$ dollars tomorrow; where $p$ is a random variable. We may assume $E(p) > 1$ still with some probability $p < 1$. 

Leverage

The Bank’s Balance Sheet today

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(1 - \alpha) L$ (reserves)</td>
<td>$L = D + \bar{K}$ (equity)</td>
</tr>
<tr>
<td>$\alpha L$ (loans and other risky investments)</td>
<td></td>
</tr>
</tbody>
</table>

The Bank’s Expected Balance Sheet tomorrow

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(1 - \alpha) L$</td>
<td>Deposits: $D$</td>
</tr>
<tr>
<td>$p(\alpha L)$</td>
<td>Capital: $\bar{K} + (p - 1) (\alpha L)$</td>
</tr>
</tbody>
</table>

- Here we see why banks hold equity: in order to be able to absorb losses (or gains) on their assets. Note that the capital tomorrow is equal to the original capital plus the realized capital gain $(p - 1)(\alpha L)$—which is a capital loss for $p < 1$.
- The bank’s *leverage ratio* is $\lambda = \frac{\text{Assets}}{\text{Capital}} = \frac{L}{\text{Capital}}$, the ratio of total assets (equal to total liabilities) to capital.
Leverage

If \( \bar{K} + (p - 1) (\alpha L) < 0 \) the bank is broke. Note that we can rewrite the condition as

\[
\bar{K} + (p - 1) (\alpha L) < 0
\]

\[
\bar{K} + p (\alpha L) - (\alpha L) < 0
\]

\[
\bar{K} + p (\alpha L) - (\alpha L) + (1 - \alpha) L < (1 - \alpha) L
\]

\[
\bar{K} + p (\alpha L) + (1 - \alpha) L < L
\]

\[
p (\alpha L) + (1 - \alpha) L < L - \bar{K}
\]

\[
p (\alpha L) + (1 - \alpha) L < D
\]

The last line says that the bank is broke when the value of assets is not enough to pay for deposits.

The possibility that the bank defaults (becomes unable to pay its debt) introduces a **non-linearity**
Leverage and the probability that the bank defaults

What is the probability that a bank will go broke?

$$\text{Prob } (\bar{K} - (1 - p) (\alpha L) < 0) = \text{Prob } \left( p < 1 - \frac{\bar{K}}{\alpha L} \right)$$

which is increasing in $\alpha$: for a given value $\bar{K}$, the higher the fraction of total assets the bank invests in the risky asset, the higher the probability it goes broke.

How do banks choose $\alpha$? They choose it so that the probability of going broke is less than or equal to some number—say 5%

$$\text{Prob } \left( p < 1 - \frac{\bar{K}}{\alpha L} \right) \leq 0.05$$

For given $\bar{K}$ the above inequality determines the value of $\alpha$. 
Leverage and Value at Risk (VAR)

| $(1 - p) \alpha L$ | is also called the bank’s **Value at Risk**. $\bar{K}$ should be large enough to absorb a loss equal to $| (1 - p) \alpha L |$ which occurs with a 5% probability
Leverage and VAR

The bank has two choices to make: $\alpha$ and $\bar{K}$. Using the expression for the leverage ratio ($\lambda = \frac{L}{K}$), the probability that a bank will go broke is

$$\text{Prob} \left( p < 1 - \frac{\bar{K}}{\alpha L} \right) = \text{Prob} \left( p < 1 - \frac{1}{\alpha \lambda} \right)$$

for given $\alpha$, the probability that a bank will go broke is an increasing function of the leverage ratio $\lambda$. To reduce $\lambda$ the bank can

- keep its assets, $L$, constant but finance them with less debt and more equity (remember, $L = \bar{K} + D$)
- keep $\bar{K}$ constant but reduce $L$ — for example reducing loans to firms and households— thus reducing $D$

for given $\alpha$, the value of $\lambda$ such that $\text{Prob}(p < 1 - \frac{1}{\alpha \lambda}) \leq 5\%$ is decreasing with $\text{Var}(p)$ \(^1\)

\(^1\)This is strictly true if the distribution of $p$ is Normal. It is not true for some other distributions.
Leverage of U.S. and European banks before the crisis

U.S. banks

- Citigorup: 19.2
- JPMorgan: 12.7
- Wells Fargo: 12.0
- Bank of America: 11.7

European banks

- UBS: 53.4
- Credit Suisse: 22.7
- Fortis: 25.5
- Dexia: 36.8
- BNP Paribas: 28.5
- Barclays: 37.8
- Royal Bank of Scotland: 21.7
- Deutsche Bank: 52.0

Banks may have a high $\lambda$ and still be safe by keeping $\alpha$ low—and indeed this was thought to be the case for European banks which own lots of "safe" government bonds. This is why Deutsche Bank was considered safe even with a value of $\lambda$ almost three times that of Citi.
In the years before the crisis macroeconomic volatility was low, thus $\text{Var}(p)$ was low.

Low $\text{Var}(p)$ meant that banks, for given $\alpha$, could afford a relatively high $\lambda$—or, for given $\lambda$, they could afford a higher $\alpha$ (they could hold a higher share of risky assets).

At the start of the crisis volatility suddenly increased and banks responded by lowering both $\lambda$ and $\alpha$.

But this takes time because raising capital and reshuffling the bank’s assets takes time.

Reducing $\alpha$ is also dangerous because:

- If the bank reduces its lending to firms (or stops lending to firms), investment will fall.
- If it sells other risky assets, such as shares, it pushes share prices down precisely at a time when the stock market (because of the crisis) is already falling. This could generate a negative leverage cycle.
Leverage Cycles and Fire Sales

Assume for simplicity $\alpha = 1$. The bank’s initial balance sheet (with leverage $= 10$) is

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>110</td>
<td>Deposits 99</td>
</tr>
<tr>
<td></td>
<td>Capital 11</td>
</tr>
</tbody>
</table>

Balance sheet after the fall in asset prices (leverage $= 10.9$)

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>109</td>
<td>Deposits 99</td>
</tr>
<tr>
<td></td>
<td>Capital 10</td>
</tr>
</tbody>
</table>

The bank can return to a leverage ratio of 10 selling assets and paying back deposits

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>Deposits 90</td>
</tr>
<tr>
<td></td>
<td>Capital 10</td>
</tr>
</tbody>
</table>

The bank ignites a fire sale: it sells assets precisely when asset prices are falling!
Raising Capital to Avoid Fire Sales

Balance sheet after the fall in asset prices (leverage = 10.9)

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>109</td>
<td>Deposits 99</td>
</tr>
<tr>
<td></td>
<td>Capital 10</td>
</tr>
</tbody>
</table>

The bank can return to a leverage ratio of 10 instead of selling assets, by raising capital

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>110</td>
<td>Deposits 99</td>
</tr>
<tr>
<td></td>
<td>Capital 11</td>
</tr>
</tbody>
</table>
How does this help us understand what happened since 2008?
Phase I: Bank Losses (writedowns) and Re-capitalizations: 08/2007 - 08/2008

Source: Bloomberg

© Bloomberg. All rights reserved. This content is excluded from our Creative Commons license. For more information, see http://ocw.mit.edu/help/faq-fair-use/.
© IMF. All rights reserved. This content is excluded from our Creative Commons license. For more information, see http://ocw.mit.edu/help/faq-fair-use/.
Source: Figure 1.3 in International Monetary Fund. "World Economic Outlook: Sustaining the Recovery." October 2009.
The Leverage of European Banks and the Crisis in the Euro Area

**European banks:**

- UBS: 53.4
- Credit Suisse: 22.7
- Fortis: 25.5
- Dexia: 36.8
- BNP Paribas: 28.5
- Barclays: 37.8
- Royal Bank of Scotland: 21.7
- Deutsche Bank: 52.0

- High leverage was not a problem so long as $\alpha$ was low because government bonds were considered safe. But when this assumption no longer held, European banks found they had far too little capital.

- This in turn made government debt more risk because the assumption was that governments would bail out the banks if they went bankrupt.
What is the financial system? What role does it play in the economy?

- The role of the financial system is to transfer the savings of households to those who need the funds to finance real economic activity, e.g. set up a new company or expand an existing one.

How can this be done? Start from the simplest case: the economy of Robinson Crusoe.
Robinson Crusoe’s Economy

Robinson has a project: a land to farm. In his economy there is no financial system, because Robinson has no one with whom to trade.

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Projects    Equity*

Projects (farming) are wholly owned by the farmer (Robinson). In this economy there is no borrowing, and no "delegation", thus no need to monitor the managers who carry out the projects.
Financial System and Delegation

- Who owns the capital rarely is the best person to run a project which uses that capital
- Modern economies work thanks to contracts that allow to delegate the running of projects to more skilful individuals
- The financial system provides the institutional framework for those contracts

Delegation to a project manager without financial intermediaries

*Firm’s balance sheet*  
*Household’s balance sheet*

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Projects</em></td>
<td><em>Debt</em></td>
<td><em>Firm’s bonds</em></td>
<td><em>Firm’s shares</em></td>
</tr>
<tr>
<td><em>Equity</em></td>
<td></td>
<td><em>Equity</em></td>
<td></td>
</tr>
</tbody>
</table>
Financial System and Delegation via Banks

**Bank’s balance sheet**

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loans to firms</td>
<td>Deposits</td>
</tr>
<tr>
<td>Other assets</td>
<td>Equity (shares issued by the bank)</td>
</tr>
</tbody>
</table>

**Firm’s balance sheet**

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Projects</td>
<td>Debt (Bonds)</td>
</tr>
<tr>
<td></td>
<td>Bank loan</td>
</tr>
<tr>
<td></td>
<td>Shares</td>
</tr>
</tbody>
</table>

**Household’s balance sheet**

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm’s bonds</td>
<td>Shares of firms and banks</td>
</tr>
<tr>
<td>Bank deposits</td>
<td>Equity</td>
</tr>
</tbody>
</table>
Why do banks exist?

- Banks (and other financial firms) exist because people in the economy are different: they have different skills and different needs—in economists’ jargon they are *heterogeneous*.

- Banks (and the financial system more generally) help solving the problems posed by this heterogeneity.
Entrepreneurs are very special individuals. Their characteristic is to have the skills necessary to turn ideas into projects and then run them. They do this borrowing from people who do not have these skills. Investors (those who own the capital) are happy to lend to entrepreneurs because they hope to participate in the returns produced by the efficient exploitation of smart ideas.

The contract between a lender and an entrepreneur is complicated:

- Entrepreneurs may not have the incentive to run their project diligently enough. Once they have raised the funds from investors they may prefer to spend time on the beach.
- Lenders (investors) cannot observe how diligently the entrepreneur is running her project, thus they can be fooled.

This "lack of trust" can be overcome if entrepreneurs risk enough of their own in the project.

Banks can facilitate the contract between entrepreneurs and lenders, monitoring the entrepreneurs.
Two examples of heterogeneity - 2: Heterogeneity wrt liquidity needs

- Agents have different preferences
  - some wish to hold very liquid assets (demand deposits)
  - other need to borrow long term, e.g. a 30-year mortgage to buy a house or to build a new plant.

- Another role banks can play is associated with the fact that they can transform maturities, i.e. borrow by issuing demand deposits and lend for 30 years.

- We shall now study two models which describe some the mechanisms underlying these two reasons why banks exist. In both, as we shall see, balance sheets are central. (Of course our list of reasons why banks exist is not exhaustive: there are a few more, like the fact that banks may be better at evaluating the firms’ projects.)

1. Entrepreneurs, banks and small investors
2. The benefits and the risks of transforming maturities and providing liquidity
1. Why do banks exist? Entrepreneurs, banks and small investors


There are 3 actors in the economy: entrepreneurs, small investors and banks

Entrepreneurs

- there are many of them; each one has
  - an idea that costs \( I \) dollars to implement
  - an amount \( A \) of cash they can dedicate to their idea, \( A < I \)
- an idea implemented today will produce tomorrow
  - \( R > 0 \) with prob \( p \)
  - \( 0 \) with prob \( (1 - p) \)
- if they don’t invest their cash in their idea, entrepreneurs can buy a safe government bond whose total return is \( 0 < \gamma I < R \)
The contract between entrepreneurs and small investors

Since $A < I$, the idea, to be implemented, needs outside funding. Assume there are only small investors. They are small in the sense that they do not have the resources to monitor how diligently the entrepreneur whom they have financed runs her project.

- Entrepreneurs can affect $p$, the probability of success, by deciding how much effort to put into running their project. This creates a moral hazard if their effort cannot be observed.
- If they put little effort they enjoy a private benefit $B$ (e.g. they spend more time on the beach, less on their project)
  - if private benefits are 0, $p = p_H$
  - if private benefits are $B$, $p = p_L < p_H$
- Small investors do not observe the entrepreneur’s effort. Their outside option (if they do not invest in the project) is buying the safe government bond.
The contract between entrepreneurs and small investors

We assume that returns are such that investing in the entrepreneur’s idea yields a higher return than investing in a safe government bond only if the entrepreneur puts in enough effort:

- \( p_H R > \gamma I \)

- \( p_L R + B < \gamma I \)
How to make sure that entrepreneurs are diligent

To make sure that she works hard and thus achieves $p_H$, small investors need to offer the entrepreneur a contract that is sufficiently attractive to induce her to work hard. Consider the following contract:

- if the project succeeds $R$ will be divided between $R_E$ for the entrepreneur and $R_S$ for the investors with $R_E$ such that the entrepreneur has an incentive to put in $p_H$

- $R_E$ must satisfy
  
  $p_H R_E \geq p_L R_E + B$

  i.e.

  $R_E \geq B / (p_H - p_L) = B / \Delta p$

- Note that, as $p_L \to p_H$, the contract becomes unfeasible because giving the entrepreneur the necessary incentive becomes impossible
Pledgeable income

Since \( R_E \geq B / \Delta p \) for the entrepreneur to be credible when he commits not to shirk, not all the income produced by the project can be pledged to outside (small) investors

\[ R_S \leq (R - B / \Delta p) < R \]

Limited pledgeability arises because of the moral hazard problem of entrepreneurs

Limited pledgeability is what makes **contract theory** (a lively branch of economics) interesting. It is also what opens up an intrusting role for financial intermediaries (banks) because sometimes they can attenuate the moral hazard problem
To have an incentive to be diligent the entrepreneur must contribute a minimum of her own to the project

- Consider the small investor. If he does not finance the project, his alternative is to buy the safe bond with a return $\gamma$. Thus he will only invest if

$$p_H R_S \geq \gamma (I - A)$$

- and since

$$R_S \leq (R - B / \Delta p)$$

- small investors will lend as long as

$$A = \bar{A}(\gamma) \geq I - [p_H / \gamma (R - B / \Delta p)]$$

i.e. unless the entrepreneur contributes a minimum amount of her own, $\bar{A}(\gamma)$, she cannot credibly commit to $p_H$

- which will be the return for small investors when they invest in the project? Competition among them will bring it down to $\gamma$, the return on their alternative option, which is investing in safe bonds
How can banks help

Banks can finance the entrepreneur’s project. But differently from small investors they can also monitor how diligently she runs it. They cannot control the entrepreneur perfectly (i.e. make sure $B = 0$) but, by spending some money, they can avoid "extreme" negligence, i.e. they can reduce the entrepreneur’s outside benefit to $b < B$. When the entrepreneur enjoys $b$ the prob of success remains $p_L$. The bank’s monitoring cost is $c > 0$.

- if the project succeeds, $R$ will be shared: $R = R_E + R_S + R_B$
- the entrepreneur must be guaranteed

$$R_E \geq \frac{b}{\Delta p}$$

where the only difference is that now $b < B$
- the bank must be guaranteed a total gross return $R_B$ large enough to make sure it has the incentive to monitor

$$p_H R_B - c \geq p_L R_B$$

i.e.

$$R_B \geq \frac{c}{\Delta p}$$
The minimum amount the bank must contribute to be credible when it says it will monitor the entrepreneur

Let $I_B$ be the amount the bank contributes to the project. How large should $I_B$ be?

To determine this value we need to start from the bank’s outside option: call $\beta$ the return the bank can obtain if it invests elsewhere in the economy. The bank must therefore be guaranteed a total net return (that is net of the monitoring cost) at least equal to $\beta I_B$, where

$$\beta I_B = p_H R_B - c = c \left(\frac{p_H}{\Delta p} - 1\right)$$

using $R_B \geq c / \Delta p$

$$I_B \geq \frac{cp_L}{\beta \Delta p}$$

For any given value of the bank’s outside option, $\beta$, this value of $I_B$ is the minimum amount the bank must contribute to be credible when it says it will monitor the entrepreneur.
The amount the bank will be asked to contribute

- Because monitoring is costly, the entrepreneur will finance through the bank as little of the project as possible, that is no more than \( I_B = \frac{c p L}{\Delta p} \). The reason is that in order to offer the bank a net return equal to \( \beta \), the entrepreneur must pay a gross return larger than \( \beta \), that is large enough to cover monitoring costs.

  - assume for instance, that the bank’s outside option—its return if it invests elsewhere in the economy—is \( \gamma \), equal to the outside option of small investors. Then the gross return the entrepreneur must pay the bank per unit it invests in the project is \( \gamma + c/I_B > \gamma \). Even in this case the entrepreneur will not fund more than \( I_B \) from the bank.

- Finally note that a bank that has no capital—and thus can contribute nothing of its own to the project—and simply finances all its loans to firms issuing deposits is useless — at least if we think that the main reason why banks exist is to monitor firms.
What is there for small investors?

When $R_E$ and $R_B$ are such that the entrepreneur has an incentive to be diligent, and the bank has an incentive to monitor, small investors get

$$R_S = \left[ R - (b + c) / \Delta p \right]$$

and must contribute $l - A - I_B$. Since their alternative remains the safe bond, they will finance the project provided

$$\gamma [l - A - I_B (\beta)] \leq p_H \left[ R - (b + c) / \Delta p \right]$$

This condition can be re-written as

$$A \geq A(\gamma, \beta) = l - I_B (\beta) - (p_H / \gamma) \left[ R - (b + c) / \Delta p \right]$$
Which Project Are Financed? (returns are net returns)
What can go wrong? (1)

A could fall (e.g. asset price shock from a bubble bursting)

Assume the entrepreneur contributes the real estate, for instance the land where the project is developed. If real estate prices fall, $A$ will fall. If it falls below $\bar{A}(\gamma)$, projects that previously could be financed only by small investors, now need a bank because the value of what the entrepreneur contributes is no longer sufficient to attract small investors.
What can go wrong? (2)

c is too small (e.g. subprimes)

Remember that $I_B \geq \frac{c \rho H}{\beta \Delta p}$. If the bank claims the cost of monitoring, $c$, is small (or underestimates the cost of monitoring) it will contribute too little to the project. Ex-post it will have no incentive to do the monitoring. In this case the entrepreneur’s private benefit will remain $B$ because

$$R_E = \frac{b}{\Delta p} < \frac{B}{\Delta p}$$

and $p = p_L$
What can go wrong? (3)

The bank may not have enough capital to credibly commit to monitor entrepreneurs. Remember that the minimum amount a bank must contribute to the project is $I_B(\beta) = \frac{c \rho H}{\beta \Delta p}$. If it commits less then $I_B(\beta)$ its return is insufficient to cover the cost of monitoring, thus the bank will not monitor.
What happened before and during the crisis?

Each one of the three things that could go wrong has gone wrong:

1. c: before the crisis banks had reduced their direct investments in the projects they had financed. They had done this selling their loans to other investors (what is called securitization, assembling a large number of mortgages and building a financial security which contains them all.) The benefit was less exposure to risk; the cost was reduced incentive to monitor.

2. $I_B(\beta)$: banks’ capital fell during the crisis. This means that banks had less capital for direct lending.

3. $A$: the fall in real estate prices and in asset prices in general, reduced the value of $A$, the resources entrepreneurs could commit to their projects.
2. Why do banks exist? Second example: heterogeneity wrt preferences

- The second role banks play is associated with the fact that they can "transform maturities", i.e. borrow short (by issuing checking accounts) and lend long (e.g. for 30 years).

- This is useful because some agents in the economy wish to hold very liquid assets (checking accounts), while other agents need to borrow long term, e.g. a 30-year mortgage to buy a house or to build a new plant. In other words people differ in their preferences as to when they want to consume.

- Without a bank it would be hard to find a mortgage. Assume all households wished to keep their savings in checking accounts: then who would buy a 30-year mortgage?
A model of maturity transformation


There are 3 periods

- $t = 0$: agents start with 1 unit of endowment each. Investment decisions are made. 2 technologies are available:
  - one delivers 1 unit of output in $t = 1$ for each unit of output invested in $t = 0$
  - the other delivers $R > 1$ units of output in $t = 2$ for each unit of output invested in $t = 0$. However, if this technology is liquidated in $t = 1$ it delivers $L < 1$ units of output

- $t = 1$ and $t = 2$: agents consume
There are 2 types of agents, and many agents of each type. The size of the population is normalized to 1.

- "patient", who consume only in $t = 2$ and nothing in $t = 1$
- "impatient" who consume all in $t = 1$
- agents learn their type only in $t = 1$ all they know in $t = 0$ is

$$\text{prob(being patient)} = \pi$$
$$\text{prob(being impatient)} = 1 - \pi$$

- of course if you knew your type in $t = 0$ you could invest everything in one technology or the other.
The world of Robinson Crusoe: no banks and no one with whom to trade

Call $I$ the amount agents invest in the technology with return $R$ at $t = 0$. Then their consumption options are

- if impatient: $c_1^A = (1 - I) + LI = 1 - (1 - L)I \leq 1$ ($= 1$ only for $I = 0$)
- if patient: $c_2^A = (1 - I) + RI = 1 + I(R - 1) \leq R$ ($= R$ only for $I = 1$)

where $A$ stands for "Autarky", Robinson’s world
Market economy

In $t = 1$ agents can trade.

- An agent who finds out he is impatient can issue a bond that promises to pay 1 unit of good in $t = 2$, sell it and eat in $t = 1$. The price of this bond is $p$. $p$ is a relative price that transforms $t = 2$ goods in $t = 1$ goods

$$p = \frac{\text{units of } t = 1 \text{ goods}}{\text{units of } t = 2 \text{ goods}}$$

- Since the impatient agent will receive $Rl$ in $t = 2$, he can trade it for $p(Rl)$ units of $t = 1$ goods

- Similarly an agent who finds out he is patient will trade $(1 - I)$ units of $t = 1$ goods, which he does not need, for $(1 - I)/p$ units of $t = 2$ goods

- Thus
  - $c_1^M = (1 - I) + pRl$ if impatient, where the superscript $M$ refers to the market equilibrium
  - $c_2^M = \frac{(1 - I)}{p} + Rl$ if patient
Market economy (cont.)

- If $1/p > R$, agents choose $I = 0$ because by trading at price $p$ they can do better than investing at return $R$. Remember

$$1/p = \frac{[\text{units of } t = 2 \text{ goods}]}{[\text{units of } t = 1 \text{ goods}]}$$

Thus

$$c^M_1 = 1, \quad c^M_2 = 1/p.$$ 

- If $1/p < R$, agents choose $I = 1$ and

$$c^M_1 = pR, \quad c^M_2 = R.$$
Neither \( p > 1/R \), nor \( p < 1/R \) are feasible because they imply that \( \{ c_1^M, c_2^M \} \) exceed the resources available to the economy.

- \( p > 1/R \implies I = 1, \ c_2^M = R \) and \( c_1^M = pR > 1 \): this is unfeasible.
- \( p < 1/R \implies I = 0, \ c_1^M = 1 \) and \( c_2^M = 1/p > R \): this is also unfeasible.
Market economy (cont.)

The only feasible equilibrium is \( p = 1 / R \) where agents choose

- \( c_1^M = 1 \) if impatient
- \( c_2^M = R \) if patient
- \( I^M \in [0, 1] \)

The last task is finding \( I^M \), the amount invested in the "long" technology. In order for these choices to be feasible \( I^M \) must be such that

\[
(1 - \pi) c_1^M = 1 - I^M \\
\pi c_2^M = RI^M
\]

So \( I^M \) must be equal to \( \pi \).
Why the market outcome is (in general) not optimal

The market equilibrium is:

- \( c_1^M = 1 \) if impatient
- \( c_2^M = R \) if patient

i.e. agents who find out they are impatient forgo \( R \) and consume \( 1 < R \).

- the market economy yields the same allocation agents would have chosen had they known their type in \( t = 0 \).
- i.e. it eliminates the inefficiency caused by uncertainty (the Autarkic equilibrium is inefficient: there is always some liquidation).
- \( \{ c_1^M = 1, \ c_2^M = R \} \) may not be the best solution. If in \( t = 0 \) agents could insure against the possibility that in \( t = 1 \) they find out they are impatient, they might wish to consume
  \[ \{ c_1^* > c_1^M = 1, \ c_2^* < c_2^M = R \} \]
  where * denotes optimal consumption levels. How could they insure? \(^2\)

\(^2\)We are assuming that agents give identical importance to consumption in the two periods, i.e. there is no discounting. In other words agents maximize \( U = (1 - \pi) u(c_1^I) + \pi u(c_2^P) \). They will wish to insure provided \( (1 - \pi) u'(1) < \pi u'(R) \) where \( u' \) is the marginal utility of consumption.
How can a bank improve upon the market outcome

- Assume we want to achieve \( \{c_1^* > 1, \ c_2^* < R, \ c_1^* < c_2^* \} \)

- in \( t = 0 \) the bank issues demand deposits: in exchange for a deposit of one unit at \( t = 0 \), agents receive either \( c_1^* \) at \( t = 1 \), or \( c_2^* \) at \( t = 2 \). To achieve this the bank, in \( t = 0 \) stores \((1 - \pi) c_1^*\) and invests \( \pi c_2^*/R \) in the technology which yields \( R \) in \( t = 2 \) (of course subject to the constraint \( (1 - \pi) c_1^* + \pi c_2^*/R = 1 \))

- the bank achieves the optimal allocation provided no individual withdraws at \( t = 1 \) unless she does not have to, i.e. unless she discovers she is impatient. No patient consumer withdraws in \( t = 1 \)

- provided \( c_1^* < c_2^* \) this assumption is not unreasonable because it would be irrational for a "patient" consumer to withdraw at \( t = 1 \) pretending he is impatient.
suppose a patient consumer anticipates that all other patient consumers will pretend they are impatient and withdraw $c_1^*$ at $t = 1$

at $t = 1$ the bank must then liquidate all its long term investment. The total amount of resources available to the bank are 

\[(1 - \pi) c_1^* + \pi \left(\frac{c_2^*}{R}\right) L < c_1^* \text{ if } \frac{c_2^*}{c_1^*} < \frac{R}{L}\]

thus for $1 < \frac{c_2^*}{c_1^*} < \frac{R}{L}$, if depositors anticipate that a large enough number of them will want to withdraw early, the bank fails

note that $1 < \frac{c_2^*}{c_1^*} < \frac{R}{L}$ always holds for $c_1^* < c_2^*$ and $c_1^* > 1$, $c_2^* < R$

bottom line: bank runs can happen because banks may be solvent but illiquid, i.e. their assets are not readily convertible into cash (here consumption).
Bank runs: when can they happen

Bank fails for sure | Bank fails only in the event of a run | Bank never fails

1 | R/L | \(C_2^*/C_1^*\)

Note: the constraint \((1 - \pi) c_1^* + \pi c_2^*/R = 1\) also needs to be satisfied.

© Source unknown. All rights reserved. This content is excluded from our Creative Commons license. For more information, see [http://ocw.mit.edu/help/faq-fair-use/](http://ocw.mit.edu/help/faq-fair-use/).
Bank runs: possible remedies

- Narrow banking. The bank invests nothing in the illiquid technology and stores everything.

- Suspension of convertibility. The bank has the option of stop paying its depositors when it runs out of cash. This means that any client who shows up "late" will see her/his deposit transformed from a demand deposit to a 2-period bond.

- Deposit insurance. The government steps in when the bank runs out of cash.
Inside and Outside Liquidity

We have already seen an example of liquidity crises in the model of "bank runs". Here we discuss a model that further highlights the importance of liquidity

[ J. Tirole, Illiquidity and all its Friends, mimeo, 2010 and Inside and Outside Liquidity, MIT Press, 2010]

- Two types of liquidity
  - **inside liquidity**: liquidity provided by markets participants with no outside intervention
  - **outside liquidity**: liquidity provided by the government through its ability to borrow against future tax revenues

- Two questions
  - Is **inside liquidity** sufficient, or, in order to prevent liquidation of otherwise profitable projects, **outside liquidity** is needed?
  - **Inside liquidity** need not only be sufficient in the aggregate. It also needs to be dispatched to those who need it
Liquidity shocks

- Three periods. The interest rate between each two subsequent periods is zero—meaning that if you do not invest in this project your alternative is a net return equal to zero.

- $t = 0$
  - an entrepreneur finances a project whose initial cost is 10, borrowing 2 from investors and contributing 8 in equity

- $t = 1$
  - with probability $1/2$ there is an overrun (a "liquidity shock"). An additional quantity, 20, of funds is needed to keep the project running. Otherwise the project is liquidated and yields no income in $t = 2$
  - with probability $1/2$, there is no overrun

- $t = 2$
  - provided the overrun, if it happened, has been covered, and provided the entrepreneur has put enough "effort" in running his project, this produces a revenue of 30
  - the revenue, 30, is then shared between the investors and the entrepreneur
Pleadgeable income (what can be promised to investors)

- In $t = 0$ the expected contribution of investors to the project is $2 + (1/2)20 = 12$. Assuming there are many investors who compete with one another, 12 is all that must be promised to them in $t = 0$ to bring them in (Remember that the interest rate is assumed to be zero)

- What goes to the entrepreneur, in case of a success, is 18. Assume that 18 is what is needed to make sure that the entrepreneur puts enough "effort" into running the project (i.e. assume 18 is the amount that solves the incentive problem we studied in the Holmstrom-Tirole model)

- The income the entrepreneur can credibly **pledge** to outside investors in $t = 0$ is thus $P = 30 - 18 = 12$ all that remains goes to the entrepreneur

- Provided investors are promised 12 in case of success, they will finance the project in $t = 0$ even if they know that (1) it might need refinancing and (2) in case of success, 18 will be turned over to the entrepreneur
Finance as you go

Now we are in $t = 1$

- if an overrun occurs, the entrepreneur will ask investors to contribute 20
- but what looked feasible in $t = 0$ (remember that investors were happy to be promised 12 knowing that they might be asked to cover an overrun) is no longer feasible in $t = 1$
- all the entrepreneur can promise is to deliver $P = 12$ in $t = 2$ if the project succeeds. If he promised more investors would understand that he would not run it efficiently
- but what is needed to keep the project going is $20 > 12$. Looked at as of $t = 1$ no investor will cover the overrun and the project will be abandoned
- trying to bring in "new" investors, offering them seniority with respect "old" investors doesn’t help. The project is abandoned even if the entrepreneur gave 0 to the old investors and promised $P$ to new ones
Insuring through a credit line

To solve the problem that arises in $t = 1$ the entrepreneur can buy insurance from a bank

- in $t = 0$ the entrepreneur contracts with a bank a credit line equal to 20. If in $t = 1$ the overrun occurs, the credit line is drawn. The bank cannot renege. It pays 20 and becomes the senior creditor: the old investors get nothing and the bank gets 12 in $t = 2$
- in $t = 0$, to commit to the credit line, the bank asks a fee equal to 4
- the bank is fine. It makes zero expected profits: it loses if the credit line is drawn, it gains if no overrun occurs

$$4 - (1/2)20 + (1/2)12 = 0$$

- in $t = 0$ investors must now contribute more than 2 (the difference between the cost of the project and the entrepreneur’s contribution). Now they must also pay the bank’s fee. Thus they must contribute $2 + 4 = 6$
- investors are happy to contribute 6 because $6 = (1/2)12$
Insuring through a credit line

- Alternatively, assume that the bank, rather than being paid a fee, is paid in shares of the firm. The firm issues 4 more shares (beyond the 2 bought by investors) so that the total capital of the project is now 6:
  - \( 4 = \frac{2}{3} \times 6 \) is the capital contributed by the bank
  - \( 2 = \frac{1}{3} \times 6 \) is the capital contributed by investors

- Expected returns (computed as of \( t = 0 \)) equal the amount of capital invested. The expected return for investors is 2, equal to the amount they paid to buy shares. The expected return of the bank is zero, since the shares were given to the bank for free (in exchange for its commitment to finance an overrun):
  - investors: \( \frac{1}{2} \times \frac{1}{3} \times 12 + \frac{1}{2} \times \frac{1}{3} \times 0 = 2 \)
  - bank: \( \frac{1}{2} \times \left[ \frac{2}{3} \times 12 \right] + \frac{1}{2} \times [-20 + 12] = 0 \)
The problem of time inconsistency

- Remember that in $t = 1$ the bank cannot renege on the credit line.
- Come $t = 1$ financing the overrun is a money-losing operation: the bank would not be willing to do it unless it is bound by a contract—an example of a situation of time inconsistency.
- Thus the credit line must be pre-arranged in $t = 0$. 
Where does the bank find the 20 it is committed to contribute to the project in the event of an overrun?

Assume that there are many firms and that their liquidity shocks are uncorrelated: half face an overrun, half do not. In other words there is no macroeconomic (or aggregate) shock.

Is there enough private liquidity in this economy to avoid inefficient liquidations, or do you need the "government" to step in to avoid liquidations?
Diversification

- Assume the number of firms is normalized to 1
- In $t = 0$ the bank receives $2/3$ of the shares of all firms—in exchange for the commitment to provide liquidity in $t = 1$ to those firms ($1/2$ of all firms) who will need it
- In $t = 1$ the bank, to honor its commitment, needs $(1/2) \times 20 = 10$
- The value of the bank’s portfolio (the value of the shares it owns) is: $[(1/2) \times 12] + [(1/2) \times (2/3) \times 12] = 10$
  - $[(1/2) \times 12]$ is the value of the bank’s shares in the firms that have a liquidity shock, where old shareholders will receive nothing and the bank will receive the entire pledgeable output;
  - $[(1/2) \times (4/6) \times 12]$ is the value of the bank’s shares in the firms that do not have a liquidity shock, and where the bank will share pledgeable output with small shareholders
- By selling its entire portfolio (assuming the existence of a market where the bank can do this) the bank obtains enough liquidity to "recapitalize" half the firms.
Diversification

If liquidity shocks are uncorrelated there is always enough inside liquidity and no project will ever be abandoned. What could go wrong?
The need for banking supervision

**Inside liquidity** need not only be sufficient in the aggregate. It also needs to be dispatched to those who need it

- this condition breaks down if banks are not perfectly diversified
- in the previous example there is only one bank: shocks cancel out in the aggregate, thus the bank is perfectly diversified
- but consider a situation where there are two firms and two banks each extending a credit line to one firm only
  - there is no aggregate liquidity shock: one firm faces an overrun and draws on its credit line; the other pays the commitment fee but does not draw because it has no overrun
  - one bank makes a profit of 4 (the commitment fee), the other a loss of 4
  - the firm which faces an overrun cannot rely on its bank to finance it and must fold up its project
- there is still enough liquidity in the aggregate, but it is not dispatched to the firm that needs it because the bank which makes a profit has no incentive to give it up, transfer it to the other so that this can deliver on its committed credit line
Liquidity crises: summing up

The lesson from this example is that inside liquidity may be insufficient to prevent liquidation of otherwise productive projects, even absent macroeconomic shocks. The government has two ways to deal with this

- it can use regulation to make sure that all banks are perfectly diversified, so that none is exposed to idiosyncratic shocks. This is the superior, though probably unrealistic solution
- alternatively, if regulation fails to achieve perfect diversification, it can step in to provide outside liquidity to those firms to which liquidity fails to be dispatched

Why can the government provide "outside liquidity" when the private sector cannot? Because the government can raise funds in $t = 1$ by taxing citizens and promising to pay them back in $t = 2$
Borrowing with collateral, leverage and the amplification and persistence of real shocks

- Why and through which channels can the financial market amplify a shock that hits the real economy, for instance a shock to productivity?

- Why does leverage arise and how does it amplify shocks?

- Why shocks do not die out immediately—i.e. they are persistent

A model of Entrepreneurs and Landlords

- The economy has a fixed quantity of land, \( \bar{k} \) that can be used to grow fruit.
- Some land is farmed by Entrepreneurs, \( (E) \), the rest of the land is farmed by Landlords, \( (L) \): \( k^L + k^E = \bar{k} \)
  - **Entrepreneurs**: their technology to grow fruit is \( y^E_t = a k^E_{t-1} \) with \( a \) (which is a measure of their productivity) constant and relatively high. Thus \( E \) have a very efficient technology with constant returns. But they own no land: to produce they need to buy the land from landlords.
  - **Landlords**: have a more traditional decreasing-returns technology: \( y^L_t = f(k^L_{t-1}) \), \( f' > 0, f'' < 0 \). Decreasing returns means that their productivity is lower the larger the portion of land they farm. Thus beyond some point they are willing to sell their land to \( E \). How much land they farm thus depends on how much they sell to \( E \). The larger is \( k^E_t \) (the land farmed by \( E \)) the smaller is the land farmed by \( L \), and thus the higher its productivity.
  - It takes one period to grow fruit: \( k^E_{t-1} \) produces fruit in \( t \).
Entrepreneurs

- E’s resources (their net worth) are not sufficient to buy the land they wish to farm.
- To buy the land E borrow from L. So L sells land to E and at the same time finances E’s purchase.
- Define q the price of land and b the amount E borrows from L.
- Loans are for one period. The gross interest rate is R: if you borrow b the following period you need to repay Rb (the interest plus the principal). R is exogenous.
- E’s budget constraint:

\[ ak_{t-1}^E - Rb_{t-1} + q_t k_{t-1}^E + b_t \geq q_t k_t^E \]

which can be re-written as

\[ q_t \left( k_t^E - k_{t-1}^E \right) \leq ak_{t-1}^E + (b_t - Rb_{t-1}) \]

the term on the left is E’s net purchase of land; the terms on the right are, respectively, E’s income and its net borrowing.
Entrepreneurs’ collateral constraint

- $L$ lend to $E$ only if the loans are guaranteed by sufficient collateral
- The collateral is land: if $E$ fails to repay the loan, $L$ keeps the land

\[ Rb_t \leq q_{t+1}k_t^E \]

- $Rb_t$ is how much $E$ needs to repay in $(t+1)$
- $q_{t+1}k_t^E$ is the value, at time $(t+1)$, when the loan is repaid, of the land used as collateral
How much will E borrow and how much land will E buy?

- E will borrow as much as they can given the constraint
  \[ b_t = \left( q_{t+1} k_t^E \right) / R \]

- Replacing \( b_t \) into \( E \)'s budget constraint we determine how much land \( E \) will buy
  \[
  (q_t - q_{t+1} / R) k_t^E = (q_t + a) k_{t-1}^E - Rb_{t-1} = ak_{t-1}^E
  \]

E spend their entire net worth, \((q_t + a) k_{t-1}^E - Rb_{t-1} = ak_{t-1}^E\), on the difference between the cost of new land, \(q_t k_t^E\), and the amount they can borrow against each unit of land they buy, \((q_{t+1} / R)\)

- \((q_t - q_{t+1} / R)\) is the downpayment \( E \) pays when buying the land: you pay \( q_t \) to buy a unit of land but you can borrow \( q_{t+1} / R \), thus what you need to pay upfront is \((q_t - q_{t+1} / R)\) per unit of land you buy. In other words

- \( E \) buys land up to the point at which the required downpayment is covered by \( E \)'s net worth. The amount of land \( E \) buys is thus
  \[
  k_t^E = ak_{t-1}^E (q_t - q_{t+1} / R)^{-1}
  \]
Borrowing and leverage

\[ k_t^E = \frac{ak_{t-1}^E}{(q_t - q_{t+1}/R)} = \frac{\text{net worth}}{(1 / \text{leverage})} = \text{leverage} \times \text{net worth} \]

- Borrowing, Entrepreneurs "leverage" their net worth buying \( k_t^E > \text{net worth} \)
- The smaller the downpayment, \( (q_t - q_{t+1}/R) \), they are required to put up, the larger the amount of land they can buy for any value of their net worth
Landlords

- $L$ farms land up to the point at which the productivity of the land they farm equals the opportunity cost—what they would earn if instead of farming they sold the land to $E$ in $t$ (and buy it $t+1$)

$$\frac{1}{R} f'\left(k_t^L\right) = q_t - q_{t+1}/R = u_t(k_t^L) = u_t(\bar{k} - k_t^E)$$

on the left is $L'$s return per unit of land they farm (discounted by $R$ because it takes one period for land to produce fruit). On the right is the alternative: instead of farming the land, $L$ can sell it in $t$ and buy it back in $t+1$ at price $q_{t+1}$ (discounted because you buy it back one period later). This alternative is the opportunity cost (also called user cost, $u_t$)

- $u_t$ plays a dual role. It is $L'$s opportunity cost of farming a unit of land, but it is also the required downpayment per unit of land $E$ buys

- This condition determines how much land $L$ farm

$$\frac{1}{R} f'(\bar{k} - k_t^E) = u_t(\bar{k} - k_t^E)$$
Steady State

- The economy is in steady state when $q_t = q_{t-1} = q^*$ and $k_t^E = k_{t-1}^E = k^E$.

- From $q_t - q_{t+1}/R = u_t(k_t^L)$ and $(q_t - q_{t+1}/R)k_t^E = ak_{t-1}^E$ we can thus determine the steady state values $q^*$ and $u^*$

\[ \frac{R-1}{R} q^* = u^* = a \]

- From the condition determining how much land $L$ farms, $\frac{1}{R} f'(k - k_t^E) = u_t$ we find $k^E^*$

- Finally, using the collateral constraint, $b_t = (q_{t+1}k_t^E) / R$.

\[ b^* = \frac{a}{R-1} k^E^* \]
Steady state

\[ f'(k^{bar} - k_t^E) = R u_t \]

© Source unknown. All rights reserved. This content is excluded from our Creative Commons license. For more information, see http://ocw.mit.edu/help/faq-fair-use/.
Leverage and Amplification of shocks

\[ k_t^E = \frac{a k_{t-1}^E}{(q_t - q_{t+1}/R)} = \text{leverage} \times \text{net worth} = \text{leverage} \times ak_{t-1}^E \]

- Assume \( E \) suffer a shock to their net worth: for instance their productivity, \( a \), falls
  - because \( \text{leverage} > 1 \), \( k_t^E \) falls by more
  - leverage amplifies shocks
- But leverage itself will change when \( a \) falls
  - assume when the shock occurs we are around the steady state
  - remember that in steady state \( q^* = \frac{R}{R-1}a \). \( q^* \) falls when \( a \) falls
  - the fall in \( q^* \) produces a capital loss on the land owned by \( E \) that reduces leverage and further amplifies the shock to \( a \)
Persistence

- As a result of the fall in \( a \), \( E \) enter the following period with lower net worth
- Thus the shock does not die out immediately: it propagates to the following period