Recitation 3: Consumer Theory and Food Stamps

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Outline For Recitation

1. Review of Income and Substitution effects and demand curves
2. Example Problem: In-Kind Transfers
   - Simple Problem: Choosing food expenditure subject to budget constraint.
   - Policy 1: A tax credit for each unit of food
   - Policy 2: Food Stamps
Review of Income and Substitution Effects and Demand Curves

DECOMPOSITION OF PRICE EFFECT INTO INCOME AND SUBSTITUTION EFFECTS

$ A \to B$: substitution effect (SE)  
$ B \to C$: income effect (IE)  
$ A \to C$: price effect (PE)

IE > 0  
IE < 0, PE = SE + IE > 0  
IE < 0, PE = SE + IE < 0
Example Problem - Setup

- 2 “goods”: Food ($F$) and all other spending ($G$)
- Income: $90
- Price of Food = $1 per unit
- Price of all of goods: $2 per unit
- Consumer preferences are Cobb-Douglas and given by:

$$U(F, G) = F^{\frac{1}{3}} G^{\frac{2}{3}}$$

- How much $F$ and $G$ will the agent consume?
Primal Problem: Choosing $F$ and $G$ to maximize utility, subject to Budget Constraint

- **Budget Constraint**: $P_G G + P_F F = I$ or $F + 2G = 90$.
- **Lagrangian is**:
  \[
  L = F^{\frac{1}{3}} G^{\frac{2}{3}} + \lambda (I - P_F F - P_G G)
  \]
- **Handy Trick** - apply monotone transformation (i.e. take logs).
- **3 first order conditions** (with respect to $F$, $G$ and $\lambda$)
  \[
  \frac{\partial L}{\partial F} = \frac{1}{3F} - P_F \lambda = 0
  \]
  \[
  \frac{\partial L}{\partial G} = \frac{2}{3G} - P_G \lambda = 0
  \]
  \[
  \frac{\partial L}{\partial \lambda} = I - P_F F - P_G G = 0
  \]
  $\rightarrow F^* = \frac{I}{3P_F} = 30 \quad G^* = \frac{2I}{3P_G} = 30$
- These are the Marshallian demands
Indirect Utility and the “Dual” Problem

- Indirect Utility: Utility consumer can attain given a budget and prices

\[ U(F^*, G^*) = \left( \frac{90}{3} \right)^\frac{1}{3} \left( \frac{90}{3} \right)^\frac{2}{3} = 30 \]

- The dual is to minimize expenditure in order to attain a given utility level

\[ L = P_F F + P_G G + \lambda \left( 30 - F^{\frac{1}{3}} G^{\frac{2}{3}} \right) \]

- 3 first order conditions (with respect to \( F \), \( G \) and \( \lambda \))

\[
\frac{\partial L}{\partial F} = P_F - \frac{\lambda}{3F} = 0 \rightarrow \lambda = 3FP_F
\]

\[
\frac{\partial L}{\partial G} = P_G - \frac{2\lambda}{3G} = 0 \rightarrow \lambda = \frac{3GP_G}{2} \rightarrow F = \frac{GP_G}{2P_F}
\]

\[
\frac{\partial L}{\partial \lambda} = 30 - F^{\frac{1}{3}} G^{\frac{2}{3}} = 0
\]

\[
\rightarrow F^*_h = 30 \left( \frac{P_G}{2P_F} \right)^\frac{2}{3} = 30 \quad G^*_h = 30 \left( \frac{2P_F}{P_G} \right)^\frac{1}{3} = 30
\]
Policy 1: Tax subsidy for Food Spending

- A subsidy of 0.50 for each dollar spent on food for these households.
- Budget constraint becomes:

$$1 - \frac{1}{2} \ F + 2G = 90$$

- Two questions:
  1. What are they going to consume now?
     - What change comes from the substitution effect?
     - What change comes from the income effect?
  2. What lump sum of income could we have given them such that they are indifferent between the lump sum and this subsidy?
Primal Problem: What are they going to consume now?

- Lagrangian is:

$$L = F^{\frac{1}{3}} G^{\frac{2}{3}} + \lambda \left( 90 - \frac{1}{2} F - 2G \right)$$

- FOCs are:

$$\frac{\partial L}{\partial F} = \frac{1}{3F} - \frac{1}{2} \lambda = 0$$
$$\frac{\partial L}{\partial G} = \frac{2}{3G} - 2\lambda = 0$$
$$\frac{\partial L}{\partial \lambda} = 90 - \frac{1}{2} F - 2G = 0$$

$$\rightarrow F^*_{\tau} = 60 \quad G^*_{\tau} = 30$$
Primal Problem: What part of this change comes from the substitution effect?

- Note that we just moved along the Marshallian (uncompensated) demand curve above.

- The substitution effect comes from moving along the Hicksian (compensated) demand curve.

\[
F_{\tau, SE \text{ only}}^* = 30 \left( \frac{P_G}{2P_F} \right)^{\frac{2}{3}} = 47.6
\]

\[
G_{\tau, SE \text{ only}}^* = 30 \left( \frac{2P_F}{P_G} \right)^{\frac{1}{3}} = 23.8
\]

- The difference between the two answers is the income effect! It increases your consumption of both.
Dual Problem: What lump-sum transfer could we give them so that they are indifferent?

- What utility level do they achieve with the tax subsidy?

\[ U(F^*_\tau, G^*_\tau) = 60^{\frac{1}{3}} \times 30^{\frac{2}{3}} \approx 37.8 \]

- So, now we want to find the income they would need to get this utility at the pre-subsidy prices. We can do this using the indirect utility function:

\[ U(1, 2, I) = \frac{I_{\text{lump sum}}^{\frac{1}{3}}}{3} \times \frac{I_{\text{lump sum}}^{\frac{2}{3}}}{3} = 37.8 \]

\[ \rightarrow I_{\text{lump sum}} = 113.4 \]

- Therefore, they need 113.4 - 90 = $23.4 to make them indifferent.

- With the subsidy, the government paid 0.50 * 60 = 30 > 23.4!
Why is the lump-sum transfer cheaper?

Why do you have to give them less? Because they have diminishing MRS!

\[ F_{\text{lump sum}}^* = \frac{113.4}{3} = 37.8 \]

\[ G_{\text{lump sum}}^* = \frac{113.4}{3} = 37.8 \]
Policy 2: Food Stamps

- What does the budget constraint look like with food stamps?
- Who is indifferent between food stamps and a lump-sum transfer?
- Who prefers a lump sum transfer to food stamps?
- Who prefers food stamps to a lump sum transfer?
- What would the budget set look like if you could sell food stamps on the black market for half their value?