Inflation traps, and rules vs. discretion

A large number of private agents play against a government.

**Government objective.** The government objective is given by the following loss function:

\[
L_{gov} = \{(y - y^*)^2 + \beta \pi^2\}
\]

where \(y\) is the equilibrium level of output, \(y^*\) is the socially optimal (first-best) level, and \(\pi\) is the inflation rate. Welfare is thus assumed to decrease with the distance between actual and first-best output, as well as with the rate of change in prices.

**Philips curve.** Let us denote the expected inflation by \(\pi_e\). Nominal wages are set so that the nominal wage \(w\) moves one to one with \(\pi_e\). The realized real wage is thus a decreasing function of the inflation surprise, namely difference between realized inflation \(\pi\) and expected inflation \(\pi_e\). Since lower labor costs stimulate output, we conclude that equilibrium output can be express as a decreasing function of the inflation surprise:

\[
y = \bar{y} + \alpha(\pi - \pi_e)
\]

This positive relation between output and inflation (or the associated negative relation between
unemployment and inflation) is commonly referred to as the Philips curve. The variable $\bar{y}$ represents the “natural” level of output—namely the level of output that obtains in equilibrium when monetary policy targets price stability or, more generally, when $\pi_e = \pi$. We assume $\bar{y} < y^*$, which means that the natural level of output is inefficiently low.

**Private agents.** The agents set prices/wages so as to minimize the losses from any subsequent inflation surprises. Since these losses are increasing in the gap between actual and expected inflation, we can represent that objective of the private agents with the following loss function:

$$L_{market} = (\pi_e - \pi)^2$$

Private agents then set $\pi_e$ so as to minimize the expectation of the above loss.

**Information and timing.** The agents move first, setting $\pi^e$ (or, equivalently, setting wages). The government moves second, choosing $\pi$ after agents have formed their inflation expectations and have fixed wages.
Best responses: government. The government choose $\pi$ so as to minimize

$$L_{gov} = \{(y - y^*)^2 + \beta \pi^2\}$$

subject to

$$y = \bar{y} + \alpha (\pi - \pi_e),$$

taking $\pi_e$ as given. Taking the FOC of this problem, and rearranging, we infer that the optimal choice of $\pi$ is given by

$$\pi = g(\pi_e) = \frac{\alpha (y^* - \bar{y}) + \alpha^2 \pi_e}{\alpha^2 + \beta}.$$

You can think of this as the best response of the government against any given $\pi_e$ chosen by the private sector. Note that $g(0) > 0$ and $g'(\pi) = \frac{\alpha^2}{\alpha^2 + \beta} \in (0, 1)$.

Best responses: private sector. The private agents choose $\pi_e$ so as to minimize

$$\mathbb{E}[L_{market}] = \mathbb{E} \left[(\pi_e - \pi)^2\right],$$

where $\mathbb{E}$ is the rational expectations operator. The best response for the private sector is thus given by

$$\pi_e = \mathbb{E}[\pi]$$
**Equilibrium.** Since private agents are rational, they can not be “fooled": they can not make systematic mistakes when predicting inflation. Along with the simplifying assumption that there is no uncertainty, this implies that $\mathbb{E}[\pi] = \pi$. That is, private agents can perfectly forecast inflation.

We infer that the equilibrium values of $\pi_e$ and $\pi$ must solve

$$
\pi = g(\pi_e) \quad \text{and} \quad \pi_e = \pi
$$

Combining, we get the following fixed-point relation for $\pi$:

$$
\pi = g(\pi) \equiv \frac{\alpha (y^* - \bar{y}) + \alpha^2 \pi}{\alpha^2 + \beta}.
$$

Recall that $g(0) > 0$ (because we have assumed $\bar{y} < y^*$) and $g'(.) \in (0,1)$. Hence, there is a unique solution to $\pi = g(\pi)$ and this is given by

$$
\pi = \bar{\pi} \equiv \frac{\alpha (y^* - \bar{y})}{\beta}.
$$

It follows that equilibrium equilibrium output is

$$
y = \bar{y}.
$$
Rules vs discretion. When the government is free to optimize over the choice of inflation, the equilibrium is given by

\[ \pi = \bar{\pi} > 0 \quad \text{and} \quad y = \bar{y} \]

Now suppose that, before the agents move and set their expectations, the government ties its hands and commits on perfect price stability (\( \pi = 0 \)) no matter what. E.g., say the government hands its monetary policy to an external authority (as it roughly happened when Greece and other Southern European countries abandoned their currencies and let Germany, in effect, set \( \pi \approx 0 \) under all circumstances). Then, the private agents will now set \( \pi_e = 0 = \pi \), implying that the equilibrium is now given by

\[ \pi = 0 \quad \text{and} \quad y = \bar{y} \]

Clearly, this equilibrium is much better: it delivers exactly the same aggregate output with less inflation!
What is going on? Ex ante, the government would like to promise low inflation. Ex post, the government has a desire to “cheat” and raise inflation, so as to reduce the “output gap”, that is, the gap between the equilibrium and the efficient level of output. But if the market expects the government to deviate, then they will adjust their expectations of inflation up to the point that the costs of extra inflation will be so high that the government will find it suboptimal to deviate. Hence, if the government has full “discretion” (freedom to choose $\pi$) and little “internal commitment”, then the economy ends up in a high-inflation trap. When this is the case, there is value for rules over discretion, for constitutional constraints on the conduct of monetary (and fiscal) policies, or for other ways to tie the government’s hands. Importantly, this is true even if the government is entirely benevolent!

This illustrates the value of commitment... of tying one’s hands... of central-bank independence... and some of the benefits of joining a currency union with a "strong" central bank.