14.05 Review

- Endogenous growth
- Ricardian equivalence and optimal taxation
- Social insurance
- Business cycles: productivity shocks
- Unemployment
- Money: neoclassical neutrality
- Money: short run real effects
Endogenous growth

- Solow model and Ramsey model:
  - Conditional convergence to steady state in the long-run
  - Growth in GDP per capita: technological progress

- Endogenous growth
  - AK model
  - Learning by doing
  - R&D
AK model

Setting as in Ramsey model

\[
\max \sum_{t=0}^{\infty} \beta^t u(c_t)
\]

\[
\text{st: } c_t + k_{t+1} \leq f(k_t) + (1 - \delta)k_t
\]

Main difference

\[
f(k_t) = Ak_t
\]

\[
r_t = f'(k_t) = A
\]

(violates Inada condition)
AK model: solution

- Euler equation:

\[
\frac{c_{t+1}}{c_t} = [\beta (1 + A - \delta)]^\theta
\]

- Guess linear policy functions: for some \( s \in (0, 1) \)

\[
c_t = (1 - s)(1 + A - \delta)k_t
\]

\[
k_{t+1} = s(1 + A - \delta)k_t
\]

- which implies

\[
\frac{c_{t+1}}{c_t} = \frac{k_{t+1}}{k_t} = \frac{y_{t+1}}{y_t} = [\beta (1 + A - \delta)]^\theta
\]
AK: solution II

- Resource constraint

\[
\frac{c_t}{k_t} + \frac{k_{t+1}}{k_t} = (1 + A - \delta)
\]

\[
\frac{c_t}{k_t} = (1 + A - \delta) - [\beta (1 + A - \delta)]^\theta = (1 - s)(1 + A - \delta)
\]

\[\implies s = \beta^\theta (1 + A - \delta)^{\theta - 1} = \beta^\theta (1 + R)^{\theta - 1}\]

- Income and substitution effects: \(\theta \leq 1\).

- Parameters affect growth rate (as opposed to Solow or Ramsey)

\[
\frac{c_{t+1}}{c_t} = \frac{k_{t+1}}{k_t} = \frac{y_{t+1}}{y_t} = [\beta (1 + A - \delta)]^\theta
\]

increasing in \(A\), \(\theta\) and \(\beta\).
Learning by doing

- Main difference: externalities.
- Output for firm $m$:

$$Y_t^m = F(K_t^m, h_t L_t^m)$$

- where

$$h_t = \eta \frac{K_t}{L_t}$$

- Important: decentralised market equilibrium difference from social planner.
Market

▶ Each firm takes $h_t$ as given, so optimization yields

$$r_t = F_1'(K_t^m, h_tL_t^m)$$

$$w_t = F_2'(K_t^m, h_tL_t^m)h_t$$

▶ then plug in $h_t = \eta k_t$

$$r_t = f'(\eta^{-1}) = A$$

▶ Euler equation

$$\frac{u'(c_t)}{u'(c_{t+1})} = \beta(1 + r_t - \delta) = \beta(1 + A - \delta)$$

▶ growth

$$\frac{c_{t+1}}{c_t} = \frac{k_{t+1}}{k_t} = \frac{y_{t+1}}{y_t} = g_t^{CE} = [\beta (1 + A - \delta)]^\theta$$
Social Planner

- Internalizes effect of capital accumulation on $h_t$. First plug in $h_t = \eta k_t$

$$y_t = \frac{Y_t}{L_t} = F(k_t, h_t) = f\left(\frac{k_t}{h_t}\right) h_t = f(\eta^{-1})\eta k_t = A^* k_t$$

$$A^* = f(\eta^{-1})\eta > A \quad \text{(why?)}$$

- So Euler condition

$$\frac{u'(c_t)}{u'(c_{t+1})} = \beta (1 + A^* - \delta)$$

- and growth

$$\frac{c_{t+1}}{c_t} = \frac{k_{t+1}}{k_t} = \frac{y_{t+1}}{y_t} = g_t^{SP} = [\beta (1 + A^* - \delta)]^\theta > g_t^{CE}$$
R&D

- Main idea: technological progress as an economic activity

\[
\max_{z_t} q(z_t) V_{t+1} - w_t z
\]

\[
\implies z_t = g \left( \frac{V_{t+1}}{w_t} \right)
\]

- Value of innovation

\[
V_{t+1} = \gamma \hat{v} A_t
\]

- Cost of innovation

\[
w_t = A_t
\]
Aggregate rate of innovation

\[ \lambda_t = q(g(\gamma \hat{\nu})) \]

Aggregate growth

\[ \frac{A_{t+1}}{A_t} = 1 + \gamma \lambda = 1 + \gamma q(g(\gamma \hat{\nu})) \]

If we increase \( \hat{\nu} \) innovation becomes more attractive. If we increase \( \gamma \) innovation becomes more attractive and, in addition, it has larger aggregate effects.

Optimal patent protection: incentives vs. externalities.
Ricardian Equivalence

- Main idea: timing of taxes has no effects on equilibrium.
- Take government expenditures as given.
- Assume taxes are not distortionary.
- No financial frictions: households can borrow and lend freely.
Ricardian equivalence

- Household intertemporal budget constraint
  \[ \sum_{t=0}^{T} q_t c_t \leq (1 + R_0) a_0 + \sum_{t=0}^{T} q_t w_t l_t - \sum_{t=0}^{T} q_t T_t \]

- and assets \( a_t = b_t + k_t \).

- Government budget constraint
  \[ \sum_{t=0}^{T} q_t g_t + (1 + R_0) b_0 = \sum_{t=0}^{T} q_t T_t \]

- therefore any tax plan that satisfies the government budget constraint leaves the household budget constraint unchanged
  \[ \sum_{t=0}^{T} q_t c_t \leq (1 + R_0) k_t + \sum_{t=0}^{T} q_t w_t l_t - \sum_{t=0}^{T} q_t g_t \]

- and so households don’t change their consumption/work/savings decisions.
Optimal taxation

- Ricardian equivalence fails if there are financial frictions (pset)
- or if taxation is distortionary (also see pset)

\[ y_t = Y - \Lambda(T_t) \]

- To simplify assume linear preferences \( u(c) = c \), so that \( (1 + R_t)\beta = 1 \), or \( q_t = \beta^t \).

- So now the problem of optimal taxation is to

\[
\max_{\{T_t\}} \sum_{t=0}^{T} \beta^t (Y - \Lambda(T_t) - g_t) \\
\text{st} : \sum_{t=0}^{T} \beta^t g_t = \sum_{t=0}^{T} \beta^t T_t
\]
Taxation smoothing

- We get taxation smoothing

\[ \Lambda'(T_t) = \lambda \implies T_t = T^* \ \forall t = 0, 1...T \]

\[ \implies T^* = (1 - \beta) \sum_{t=0}^{T} \beta^t g_t \]

- Permanent increase in \( g \)
- Transitory increase in \( g \)
Social insurance

- Main idea: taxation and redistribution can provide ex-ante insurance (before we know whether we will be lucky/succesful).

\[ u_i = - \exp \left\{ - \left( c_i - \frac{n_i^{1+\epsilon}}{1 + \epsilon} \right) \right\} \]

\[ c_i = (1 - \tau) y_i + T = (1 - \tau) (n_i + \nu_i) + T \]

- So FOC

\[ n_i = (1 - \tau)^{\frac{1}{\epsilon}} \]

- to simplify \( \epsilon = 1 \) so

\[ n_i = (1 - \tau) \]
Social insurance II

- Government budget

\[ T = \tau \int y_i \, di = \tau (1 - \tau) \]

- Then agent’s utility

\[ u_i = - \exp \left\{ - \left( \frac{1}{2} (1 - \tau)^2 + (1 - \tau) \nu_i + \tau (1 - \tau) \right) \right\} \]

- Maximize its expectation (before knowing \( \nu_i \))

\[ \max \mathbb{E} [u_i] = - \exp \left\{ - \left( \frac{1}{2} (1 - \tau)^2 - \frac{1}{2} (1 - \tau)^2 \sigma^2 + \tau (1 - \tau) \right) \right\} \]

- Optimal \( \tau \) increases with \( \sigma \) (agents don’t like risk).
Use graph of labor market and market for capital services: $\frac{w}{P}$ and $\frac{R}{P}$

Increase in productivity:

- higher real wage $\frac{w}{P}$, and hours worked $L$.
- higher rental price $\frac{R}{P}$ and capital utilization $\kappa K$.

Consumption: income vs substitution

- Permanent shock $C$ and $I$ both go up, so $K$ goes up as well.
- transitory shock: $C$ could go up or down, $I$ goes up, and hence $K$ as well
Search model of unemployment

- Job finding (50% of $U$): workers receive wage offers $\frac{w}{P}$, and have a reservation wage $\omega$.
  - from unemployment income
  - from option value of waiting for a better offer
  - Productivity shock improves offers more than the reservation wage: more job finding.
- Job-separation (3% of $L$)
- Natural unemployment rate $u = \frac{U}{U+L}$
  
  $\phi U = \sigma L = \sigma(U + L - U)$
  
  $(\phi + \sigma)U = \sigma(U + L)$
  
  $u = \frac{\sigma}{\phi + \sigma}$

- Job vacancies procyclical.
Money neutrality in neoclassical model

- Dichotomy: real variables independent of nominal

\[ \frac{M}{P} = L(Y, i) \]

\[ i = r + \pi \]

- Money neutrality: permanent change in \( M \) \( \Longrightarrow \) permanent proportional change in \( P \).

- Constant growth rate \( \mu \) for \( M \) leads to constant inflation rate

\[ \pi = \mu \]

\[ \frac{M}{P} = L(Y, r + \pi) \]

- So changes in \( \mu \) affect \( P \) right away.
Misperception model

- Money has real effects in the short-run
  - misperception model (here)
  - new-keynesian model
- average price level is $P$, but workers think it’s $P^e$, so they supply labor according to
  \[
  \frac{W}{P^e} = \frac{W}{P} \left( \frac{P}{P^e} \right)
  \]
- Long-run: $P^e = P$.
- Short-run: $P^e$ fixed... then it adjusts towards $P$. 
$M$ has real effects in the short run

- Increase in $M$ to $(1 + \Delta) M_0$ should increase prices to $(1 + \Delta) P_0$
- but since $P^e = P_0$ is fixed, employment $L$ and hence output $Y = F(K, L)$ go up.
- So prices don’t go up by as much, in the short run $P_0 < P_{SR} < (1 + \Delta) P_0$
- Eventually workers adjust their $P^e = P$, employment and output fall back to their long-run level, and $P \to (1 + \Delta) P_0$
- money still neutral in the long-run
- only unexpected changes in $M$ have real effects
Important papers

► Acemoglu et al.: The colonical origins of comparative economic development
  ▶ Main idea: Institutions are really important
  ▶ To show this, look at institutions built by European powers in different colonies
  ▶ where settler mortality was low, they established lots of Europeans and good institutions
  ▶ where settler mortality was high, they couldn’t, so they set up (bad) extractive institutions.

► Angeletos and Alessina:
  ▶ if people think income is luck, they tax, so work doesn’t pay off and income is mostly luck.
  ▶ if instead they think its effort, they don’t tax, so work pays off and it’s mostly effort.
  ▶ multiple equilibria: Europe vs USA
14.05 Intermediate Macroeconomics
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