Road Map

1. Definitions
2. Single-deviation principle
3. Examples
Infinitely repeated Games with observable actions

- $T = \{0, 1, 2, \ldots, t, \ldots\}$
- $G$ = “stage game” = a finite game
- At each $t$ in $T$, $G$ is played, and players remember which actions taken before $t$;
- Payoffs = Discounted sum of payoffs in the stage game.
- Call this game $G(T)$. 
Infinitely-repeated PD

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>5,5</td>
<td>0,6</td>
</tr>
<tr>
<td>D</td>
<td>6,0</td>
<td>1,1</td>
</tr>
</tbody>
</table>

Some Strategies:

**Grim Strategy**: Play C at $t=0$; thereafter play C iff D has never played before.

**Tit for Tat**: Start with C; thereafter, play what the other player played in the previous round.

**Naively Cooperate**: always play C.

\[
C \Rightarrow 5 + 5d \cdot V_d \\
C \Rightarrow 6 + d \cdot V_D \\
C \iff d > 1/5.
\]
Definitions

The *Present Value* of a given payoff stream $\pi = (\pi_0, \pi_1, \ldots, \pi_t, \ldots)$ is

$$PV(\pi; \delta) = \pi_0 + \delta \pi_1 + \ldots + \delta^t \pi_t + \ldots$$

The *Average Value* of a given payoff stream $\pi$ is

$$(1-\delta)PV(\pi; \delta) = (1-\delta)(\pi_0 + \delta \pi_1 + \ldots + \delta^t \pi_t + \ldots)$$

The *Present Value* of a given payoff stream $\pi$ *at* $t$ is

$$PV_t(\pi; \delta) = \pi_t + \delta \pi_{t+1} + \ldots + \delta^s \pi_{t+s} + \ldots$$

A *history* is a sequence of past observed plays
e.g. (C,D), (C,C), (D,D), (D,D) (C,C)
Recall: Single-Deviation Principle

- $s = (s_1, s_2, \ldots, s_n)$ is a SPE
- $\Leftrightarrow$ it passes the following test
- for each information set, where a player $i$ moves,
  - fix the other players’ strategies as in $s$,
  - fix the moves of $i$ at other information sets as in $s$;
  - then $i$ cannot improve her conditional payoff at the information set by deviating from $s_i$ at the information set only.
Single-Deviation Principle: Reduced Game

- $s = (s_1, s_2, \ldots, s_n)$, date $t$, and history $h$ fixed
- Reduced Game: For each terminal node $a$ of the stage game at $t$,
  - assume that $s$ is played from $t+1$ on given $(h, a)$
  - write $PV(h, a, s, t+1)$ for present value at $t+1$
  - Define utility of each player $i$ at the terminal node $a$ as $u_i(a) + \delta PV(h, a, s, t+1)$
- Single-Deviation Principle: $s$ is SPE $\Leftrightarrow$ for every $h$ and $t$, $s$ gives a SPE in the reduced game
Reduced Game for (Grim,Grim)

With previous defection:

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>5 +δ/(1−δ)</td>
<td>0 +δ/(1−δ)</td>
</tr>
<tr>
<td></td>
<td>5 +δ/(1−δ)</td>
<td>6 +δ/(1−δ)</td>
</tr>
<tr>
<td>D</td>
<td>6 +δ/(1−δ)</td>
<td>1 +δ/(1−δ)</td>
</tr>
<tr>
<td></td>
<td>0 +δ/(1−δ)</td>
<td>1 +δ/(1−δ)</td>
</tr>
</tbody>
</table>

Without previous defection:

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>5 +5δ/(1−δ)</td>
<td>0 +δ/(1−δ)</td>
</tr>
<tr>
<td></td>
<td>5 +5δ/(1−δ)</td>
<td>6 +δ/(1−δ)</td>
</tr>
<tr>
<td>D</td>
<td>6 +δ/(1−δ)</td>
<td>1 +δ/(1−δ)</td>
</tr>
<tr>
<td></td>
<td>0 +δ/(1−δ)</td>
<td>1 +δ/(1−δ)</td>
</tr>
</tbody>
</table>

C=> 5+5dVd
C=> 6+dVD
C <=> d>1/5.
Is (Tit-for-tat,Tit-for-tat) a SPE?

- **Tit-for-Tat**: Start with C; thereafter, play what the other player played in the previous round.
- No!
- Consider (C,C) at $t-1$ and Player 1.
  - $C => 5/(1-\delta)$
  - $D => 6/(1-\delta^2)$
  - No Deviation $\Leftrightarrow \delta \geq 1/5$.
- Consider (C,D) at $t-$ and Player 1.
  - $C => 5/(1-\delta)$
  - $D => 6/(1-\delta^2)$
  - No Deviation $\Leftrightarrow \delta \leq 1/5$.
- Not SPE if $\delta \neq 1/5$. 
Modified Tit-for-Tat

Start with C; if any player plays D when the previous play is (C,C), play D in the next period, then switch back to C.
Infinite-period entry deterrence

Strategy of Entrant:
Enter iff Accomodated before.

Strategy of Incumbent:
Accommodate iff accomodated before.
Reduced Games

Accommodated before:

1 Enter 2 Acc. → $1 + \frac{\delta}{(1-\delta)}$

Not Accommodated before:

1 Enter 2 Acc. → $1 + \frac{\delta}{(1-\delta)}$

X → Fight

$0 + \frac{\delta}{(1-\delta)} \quad -1 + \frac{\delta}{(1-\delta)}$

$2 + \frac{\delta}{(1-\delta)} \quad -1 + \frac{\delta}{(1-\delta)}$

$2 + 2\frac{\delta}{(1-\delta)} \quad -1 + 2\frac{\delta}{(1-\delta)}$
14.12 Economic Applications of Game Theory
Fall 2012

For information about citing these materials or our Terms of Use, visit: http://ocw.mit.edu/terms.