Taxing externalities with measurable pollution

Atmosphere:

\[ A = \sum_i a^j x_0^i \]  

(1)

The only route for externalities is through \( A \). Note \( A \) is the same for all consumers. Note linearity is not important. This could be done with a vector of different atmospheres, e.g., for different locations.

Pareto optimality assuming linear technology with fixed producer prices \( p \):

\[
\text{Max } \sum_h \alpha^h u^h \left[ x_0^h, x^h, A \right] \\
\text{s.t. } \sum_h \left( p_0 x_0^h + p x^h \right) = R
\]

(2)

First order conditions with respect to \( x_i^h, x_0^h \):

\[
\alpha^h \frac{\partial u^h}{\partial x_i} = \lambda p_i \quad h = 1, 2, \ldots, H; \ i = 1, 2, \ldots, N
\]

(3)

\[
\alpha^h \frac{\partial u^h}{\partial x_0} + a^h \sum_k \alpha^k \frac{\partial u^k}{\partial A} = \lambda p_0
\]

(4)

Substituting from (3) in (4)

\[
\frac{\partial u^h}{\partial x_0^h} / \frac{\partial u^h}{\partial x_i^h} = \frac{p_0}{p_i} - a^h \sum_k \frac{\partial u^k}{\partial A} / \frac{\partial x_i^k}{\partial x_i^h}
\]

(5)
I. If we can measure the pollution contribution, $a^h x_0^h$, we can decentralize the PO by pricing pollution, although prices might need to vary by person.

The consumer problem becomes:

$$\text{Max } u^h[x_0^h, x^h, A]$$

$$\text{s.t. } p_0 x_0 + p_x x + t^h a^h x_0 = I^h$$

(6)

First order conditions

$$\frac{\partial u^h / \partial x_0^h + a^h \partial u^h / \partial A}{\partial u^h / \partial x^h} = \frac{p_0 + t^h a^h}{p_1}$$

(7)

This will support the PO provided

$$\frac{t^h}{p_1} = \frac{\partial u^h / \partial A}{\partial u^h / \partial x^h} - \sum_k \frac{\partial u^k / \partial A}{\partial u^k / \partial x^k}$$

(8)

If individuals ignore their own feedback to the atmosphere, individual choice now has FOC:

$$\frac{\partial u^h / \partial x_0^h}{\partial u^h / \partial x^h} = \frac{p_0 + t^h a^h}{p_1}$$

(9)

This allows support for the PO with uniform taxes

$$\frac{t}{p_1} = \sum_k \frac{\partial u^k / \partial A}{\partial u^k / \partial x^k}$$

(10)

Note this extends to a vector of (local) atmospheres and more than one externality generating good, provided pricing distinguishes each atmosphere.
II. Alternatively, assuming consumers ignore the feedback on self through \( A \), decentralization can be approached by taxing good zero.

\[
\begin{align*}
\text{Max} & \quad u^h \left[ x^h_0, x^h, A \right] \\
\text{s.t.} & \quad (p_0 + t^h) x^h_0 + p.x^h = I^h
\end{align*}
\]

First order condition:

\[
\frac{\partial u^h}{\partial x^h_0} = \frac{p_0 + t^h}{p_i}
\]

\[
t^h = a^h p_1 \sum_k \frac{\partial u^k}{\partial x^k_i} \frac{\partial A}{\partial x^k_i}
\]

This relies on the lack of choice in how the good is consumed, with different choices resulting in different levels of pollution.