Handout on contingent commodities in an exchange economy

Consumer choice

\[ \text{Max } \sum_s \pi^h_s u^h(x^h_s) \]
\[ \text{s.t. } \sum_s p_s x^h_s = \sum_s p_s e^h_s \]  
(1)

FOC

\[ \pi^h_s u^h(x^h_s) = \lambda^h p_s \]  
(2)

Market clearance

\[ \sum_h x^h_j = \sum_h e^h_j \]  
(3)
I. Everyone the same: \( \pi^h_s = \pi_s; \ u^h(x) = u(x); \ e^h_s = e_s \)

\[
\begin{align*}
\lambda p_s &= \pi_s u'(e_s) \\
\lambda p(s) &= \pi(s) u'(e(s))
\end{align*}
\]  

(4)

Logarithmic derivative

\[
\frac{p'(s)}{p(s)} = \frac{\pi'(s)}{\pi(s)} + \frac{u''(e(s))}{u'(e(s))} e'(s)
\]  

(5)

II. Same probabilities, different utilities: \( \pi^h_s = \pi_s \)

\[
\frac{u''(x^h_s)}{u'^h(x^h_s)} x'^h(s) = \frac{p'(s)}{p(s)} - \frac{\pi'(s)}{\pi(s)}
\]  

(6)
III. Same utility (logarithmic), different probabilities, no aggregate variation

FOC

\[ \pi_s^h u'(x_s^h) = \lambda^h p_s \]  
(7)

\[ \pi_s^h / x_s^h = \lambda^h p_s \]  
(8)

Market clearance

\[ \sum_h x_s^h = \sum_h e_s^h = E \]  
(9)

IV. General utility (Yaari)  \( u^h(x_1, ..., x_S) \)

\[ \frac{\partial u^h}{\partial x_s^h} / \frac{\partial u^h}{\partial x_s} = \frac{\partial u^{h'}}{\partial x_s^h} / \frac{\partial u^{h'}}{\partial x_s} \]  
(10)