Handout on stock market model

2 periods, 1 good per period, multiplicative uncertainty
only assets: safe real bond, shares

Notation:
- $x_0$: consumption in period 0
- $x_{1s}$: consumption in period 1 in state $s$
- $\pi^h_s$: subjective probability of household $h$ for state $s$
- $b$: bonds
- $q$: price of bonds
- $D^f_s$: dividends of firm $f$ in state $s$
- $q^f$: price of all shares in firm $f$
- $\theta^h_f$: fraction of firm $f$ owned by household $h$ after purchase
- $\theta^e_h$: fraction of firm $f$ in initial endowment of household $h$
- $k^f$: input of firm $f$

Consumer choice:

\[
\max_{x_0,x_1} \sum_s \pi^h_s u^h(x_0, x_{1s}) \\
\text{s.t. } x_0 + q b + \sum_f q^f \theta^h_f = e^h_0 + \sum_f q^f \theta^e_f \\
\quad x_{1s} = b + \sum_f \theta^h_f D^f_s + e^h_s \quad \forall s
\]  

Dividend payment:

\[
y^f_s = a^f_s g^f(k^f) \\
D^f_s = a^f_s g^f(k^f) - \frac{k^f}{q}
\]
Firm maximization of stock market value:
\[
\max_k q^f \\
\text{s.t. } \text{"competitive perceptions"}
\]  

\[
\left(q^f + k^f\right) \frac{g^{lf}\left(k^f\right)}{g^f\left(k^f\right)} = 1
\]  

Market clearance
\[
\sum_h x^h_0 = \sum_h e^h_0 - \sum_f k^f
\]  

Conditions for constrained Pareto optimality:
\[
\max \sum_s \pi_s^l u^l\left(x^l_0, x^l_{1s}\right) \\
\text{s.t. } \sum_s \pi_s^h u^h\left(x^h_0, x^h_{1s}\right) = \Pi^h, \quad h = 2, \ldots, H \\
\sum_h x^h_0 = \sum_h e^h_0 - \sum_f k^f \\
x^h_{1s} = e^h_{1s} + \sum_f \mu^h_{fs} y^f_{1s} + z^h \\
\sum_h z^h = 0 \\
\sum_h \mu^h_{fs} = 1
\]