Handout contrasting stock market (SM) and complete contingent commodity (CCC) models

2 periods, 1 good per period, multiplicative uncertainty
no-bankruptcy and no-short-sales constraints not binding
second period has \( S \) states

SM: only assets: safe real bond, shares  
CCC: all commodities

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>( b )</td>
<td>bonds</td>
</tr>
<tr>
<td>( p_0 )</td>
<td>price of good in period 0</td>
</tr>
<tr>
<td>( q )</td>
<td>price of bonds</td>
</tr>
<tr>
<td>( p_s )</td>
<td>price of good in state ( s ) in period 1</td>
</tr>
<tr>
<td>( D_f^s )</td>
<td>dividends of firm ( f ) in state ( s )</td>
</tr>
<tr>
<td>( Q_f )</td>
<td>profit of firm ( f )</td>
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<tr>
<td>( q_f^h )</td>
<td>price of all shares in firm ( f )</td>
</tr>
<tr>
<td>( \theta_f^h )</td>
<td>fraction of firm ( f ) purchased by household ( h )</td>
</tr>
<tr>
<td>( \theta_f^{eh} )</td>
<td>fraction of firm ( f ) in initial endowment of household ( h )</td>
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Consumer choice:

\[
\begin{align*}
\max_{x_0, x_1} & \quad \sum_s \pi_s^h u^h(x_0, x_1) \\
\text{s.t.} & \quad x_0 + q b + \sum_f q_f^h \theta_f^h = e_0^h + \sum_f q_f^{eh} \theta_f^{eh} \quad \sum_f q_f^h D_f^s + e_s^h \forall s
\end{align*}
\]

SM has \( S + 1 \) budget constraints, while CCC has 1.

\[
\begin{align*}
D_f^s &= a_f^s g^f(k_f) - \frac{k_f}{q} \\
Q_f &= \sum_s p_s a_f^s g^f(k_f) - p_0 k_f
\end{align*}
\]

Firm choice:

\[
\begin{align*}
\max_k q_f^h & \quad \max_k Q_f \\
\text{s.t.} & \quad "competitive perceptions" \quad \text{s.t.} \quad "competitive perceptions"
\end{align*}
\]
First order conditions:

\[
(q^f + k^f) \frac{g^{ff}(k^f)}{g^f(k^f)} = 1 \quad \sum_s p_{1s} a_s^f g^{ff}(k^f) = p_0 \quad (4)
\]

Market clearance:

\[
\sum_h x_0^h = \sum_h e_0^h - \sum_f k^f \quad \sum_h x_0^h = \sum_h e_0^h - \sum_f k^f \quad (5a)
\]

\[
\sum_h \theta^h_f = 1 \quad f=1,\ldots,F \quad \sum_h x_{1s}^h = \sum_h e_0^h + \sum_f a_s^f g^f(k^f) \quad s=1,\ldots,S \quad (5b)
\]

\[
\sum_h q_b^h = \sum_f k^f \quad (5c)
\]

Note that Walras Law gives (5c).

Constrained Pareto optimality:

\[
\max \sum_s \pi_s^u l \left( x_0^s, x_{1s}^l \right) \quad \text{s.t.} \quad \sum_s \pi_s^h u^h \left( x_0^h, x_{1s}^h \right) = v^h, \quad h = 2,\ldots,H
\]

\[
\sum_h x_0^h = \sum_h e_0^h - \sum_f k^f
\]

\[
x_{1s}^h = e_0^h + \sum_f \mu^h_f a_s^f g^f(k^f) + z^h
\]

\[
\sum_h \mu^h_f = 1
\]

\[
\sum_h z^h = 0
\]

Pareto optimality:

\[
\max \sum_s \pi_s^u l \left( x_0^s, x_{1s}^l \right) \quad \text{s.t.} \quad \sum_s \pi_s^h u^h \left( x_0^h, x_{1s}^h \right) = v^h, \quad h = 2,\ldots,H
\]

\[
\sum_h x_0^h = \sum_h e_0^h - \sum_f k^f
\]

\[
\sum_h x_{1s}^h = \sum_h e_0^h + \sum_f a_s^f g^f(k^f) \quad (6)
\]