14.123 Microeconomics III—Problem Set 1

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Instructions. Each question is 33 points. Make the necessary technical assumptions as you need them. Good Luck!

1. Consider the following game

\[
\begin{array}{cccc}
 w & x & y & z \\
 a & 3,2 & 0,0 & 0,0 & 1,1 \\
b & 0,0 & 2,3 & 0,0 & 1,1 \\
c & 0,0 & 0,0 & 0,0 & -1,-1 \\
d & 1,1 & 1,1 & -1,-1 & 0,0 \\
\end{array}
\]

(a) Compute the set of rationalizable strategies.
(b) Compute the set of correlated equilibrium distributions.
(c) Identify a correlated equilibrium that is not a Nash equilibrium.

2. This question asks you to establish the formal link between correlated equilibrium and Bayesian Nash equilibrium. Assuming everything is finite, consider a game \( G = (N, S, u) \).

(a) For any given (common-prior) information structure \( (\Omega, I, p) \), find a type space \( (T, p') \) where the types do not affect the the payoffs in \( G \) and a one-to-one mapping \( \tau_i \) between the information cells \( I_i(\omega) \) and types \( \tau_i(I_i(\omega)) \in T_i \) (for all \( i \in N \)), such that an adapted strategy profile \( s = (s_1, \ldots, s_n) \) w.r.t. \( (\Omega, I, p) \) is a correlated equilibrium if and only if \( s \circ \tau^{-1} \) is a Bayesian Nash equilibrium of \( (G, T, p') \). [Here, \( s \circ \tau^{-1} = (s_1 \circ \tau_1^{-1}, \ldots, s_n \circ \tau_n^{-1}) \) is such that, for every type \( t_i \), \( s_i \circ \tau_i^{-1}(t_i) = s_i(\omega) \) for some \( \omega \) with \( \tau_i(I_i(\omega)) = t_i \).]

(b) For any type space \( (T, p') \) where the types do not affect the the payoffs in \( G \), find a information structure \( (\Omega, I, p) \) and a one-to-one mapping \( w : T \rightarrow \Omega \) such that \( s = (s_1, \ldots, s_n) \) is a Bayesian Nash equilibrium of \( (G, T, p') \) if and only if \( s \circ w^{-1} \) is a correlated equilibrium.

3. For any given game \( G = (N, S, u) \), a set \( Z = Z_1 \times \cdots \times Z_n \subseteq S \) is said to be closed under rational behavior if for every \( i \in N \), \( z_i \in Z_i \), there exists \( \mu \in \Delta(Z_{-i}) \) such that \( z_i \in \arg \max_{s_i} u_i(s_i, \mu) \).

(a) Show that if \( Z \) is closed under rational behavior, then \( Z \subseteq S^\infty \).
(b) Show that for any family of sets \( Z^\alpha \) that are closed under rational behavior, the set \( Z = (\cup_\alpha Z^\alpha_1) \times \cdots \times (\cup_\alpha Z^\alpha_n) \) is closed under rational behavior. Conclude that the largest set \( Z^* \) that is closed under rational behavior exists.
(c) Show that \( Z^* = S^\infty \).