1. I work with certainty equivalent as in the lecture.

1.1 Certainty equivalent of the individual’s continuation utility is $(\mu - \frac{1}{2} \alpha \sigma^2)(T - t + 1) + X_{i_1} + \cdots + X_{i(t-1)}$, and the individual is willing to sell the asset for price $P_t = (\mu - \frac{1}{2} \alpha \sigma^2)(T - t + 1) + X_{i_1} + \cdots + X_{i(t-1)}$. Expected value of the asset to the company is $\mu(T - t + 1) + X_{i_1} + \cdots + X_{i(t-1)} > P_t$ and so, the company should make such offer.

1.2 The company buys all assets at date 1 paying $\Pi_1 = (\mu - \frac{1}{2} \alpha \sigma^2)Tn$ and the return is $\sum_i Y_i$. Certainty equivalent of the dividend of the new individual equals $\mu T - \frac{T}{2n} \alpha \sigma^2 - \frac{\Pi_1}{n} = \frac{1}{2} \alpha \sigma^2 (1 - \frac{1}{n})$, which is the maximal price that a new individual is willing to pay for a $1/n$ share.

1.3 Suppose that company could buy the assets only starting from date $t = 2$. Then it pays for the assets price $\Pi_2 = (\mu - \frac{1}{2} \alpha \sigma^2)(T - 1)n + \sum_i X_{i_1}$. Certainty equivalent of the dividend equals $\frac{1}{2} \alpha \sigma^2 (T - 1)(1 - \frac{1}{n})$.

1.4 A new individual is willing to pay less for the share of the company with restricted trades, since he expects that the assets will be purchased by the company at $t = 2$ at a higher price.

2. I start by setting up the general problem. The Bellman equation for the problem is

$$V(\omega) = \max_{s \in [0, \omega]} u(\omega - s) + \delta(1 - \rho)\mathbb{E}[V(sr)].$$

The first-order condition for this problem together with the envelope condition $V'(\omega) = u'(\omega - s)$ gives

$$u'(\omega - s) = \delta(1 - p)\mathbb{E}[ru'(rs - s_+)],$$

where $s_+$ is the savings in the next round.

2.1 In (1), set $r = 1$, $(c_{0*})^{-\rho} = \delta(1 - p)(\omega - c_{0*})^{-\rho}$ or $c_{0*} = \frac{1}{1 + \delta(1-p)\rho \omega}$ and $c_{1*}^{\star} = \frac{\delta^{1/(1-p)}(1-p)^{1/p}}{1 + \delta^{1/(1-p)}(1-p)^{1/p}} \omega$. 

1
2.2 In (1), set $r = 1$, $(c_t^*)^{-\rho} = \delta(1-p)(c_{t+1}^*)^{-\rho}$ or $c_{t+1}^* = c_t^* \delta^{1/\rho}(1-p)^{1/\rho} = c_0^* \delta^{(t+1)/\rho}(1-p)^{(t+1)/\rho}$. Therefore, $c_0^* = (1 - \delta^{1/\rho}(1-p)^{1/\rho}) \omega$.

2.3 For $\rho = 1$, $c_{t+1}^* = c_0^* \delta^{t+1}(1-p)^{t+1}$ and $c_0^* = (1 - \delta(1-p)) \omega$.

2.4 We guess that solution takes form $c = \alpha \omega$ and plug it into (1) to get

$$\frac{1}{\alpha c} = \delta(1-p) \mathbb{E} \left[ \frac{r}{\alpha r \omega (1 - \alpha)} \right],$$

and so, $c_t^* = (1 - \delta(1-p)) \omega_t$ where $\omega_t = r_t(\omega_{t-1} - c_{t-1})$ and $\omega_0 = \omega$.

3. I denote by $\succeq$ first-order stochastic dominance relationship, and by $\succeq$ second-order stochastic dominance relationship.

3.1 True. $P(g(X) \leq t) = P(X \leq g^{-1}(t)) \leq P(Y \leq g^{-1}(t)) = P(g(Y) \leq t)$.

3.2 False. Consider $X = \frac{1}{2}$ and $Y$ is uniform on $[0,1]$. Then $X \succeq Y$. Consider $g(t) = t^2$ and $u(t) = t$, then $\mathbb{E} u(g(Y)) = \mathbb{E} g(Y) > g(\mathbb{E} Y) = g(X) = \mathbb{E} u(g(X))$.

3.3 False. Consider the following counter-example. Let $\alpha = \frac{1}{2}$, $X$ is uniform on $[0,1]$ and $Y = 1 - X$. Since $X$ and $Y$ have the same distribution, $X \succeq Y$. However, $\frac{X+Y}{2} = \frac{1}{2}$ and $X$ are not ordered according to $\succeq$.

4. Denote the share invested in asset $i = 1, 2$ by $z_i$. Optimal portfolio solves the problem

$$\max_{z_1, z_2} \mathbb{E} - e^{-\alpha (\omega + z_1(X-1) + z_2(Y-1))} = \max_{z_1, z_2} \omega + z_1(\mu - 1) - \frac{z_1^2}{2} \alpha \sigma^2 + z_2(2\mu - 1) - \frac{z_2^2}{2} \alpha \sigma^2$$

which has solution $z_1 = \frac{1}{\alpha} \frac{\mu - 1}{\sigma^2}$ and $z_2 = \frac{1}{\alpha} \frac{2\mu - 1}{\sigma^2}$.