Question 1

Apply the forward-induction iterative elimination procedure described below to the following game. Two players, 1 and 2, have to play the Battle of the Sexes (BoS) game with the following payoff matrix

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(3,1)</td>
<td>($\varepsilon, \varepsilon$)</td>
</tr>
<tr>
<td>B</td>
<td>($\varepsilon, \varepsilon$)</td>
<td>(1,3)</td>
</tr>
</tbody>
</table>

where $\varepsilon$ is a small but positive number. Before playing this game, player 1 first decides whether to burn a util; if he does so, his payoffs decrease by 1 at each action profile in BoS. Then player 2 observes player 1’s decision and decides whether to burn a util herself, which would reduce her payoffs by 1 for each action profile in BoS. After both players observe each other’s burning decisions, they play BoS.

The iterative procedure is as follows. Let $S_i$ be player $i$’s pure strategy space.

- For step $t = 0$, set $S_i^0 = S_i$.
- At any step $t \geq 1$, for each player $i$ and information set $h$ of $i$, let $\Delta^t_i(h)$ be the set of all beliefs $\mu_i(h) \in \Delta(S_{-i}^t)$ such that $\mu_i(s_{-i}|h) > 0$ only if $h$ can be reached by some strategy in $S_i \times S_{-i}^t$. For each $s_i \in S_i^t$, eliminate $s_i$ if there exists an information set $h$ for player $i$ such that $s_i$ is not sequentially rational at $h$ with respect to any belief $\mu_i(h) \in \Delta^t_i(h)$. Let $S_i^{t+1}$ denote the set of remaining strategies.
- Iterate until no further elimination is possible.
Question 2

(a) Consider the repeated game $RG(\delta)$, where the stage game is matching pennies:

<table>
<thead>
<tr>
<th></th>
<th>$H$</th>
<th>$T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H$</td>
<td>1,-1</td>
<td>-1,1</td>
</tr>
<tr>
<td>$T$</td>
<td>-1,1</td>
<td>1,-1</td>
</tr>
</tbody>
</table>

For any discount factor $\delta \in (0,1)$, find all the subgame-perfect equilibria of the repeated game.

(b) A game $G = (N, A, u)$ is said to be a zero-sum game if $\sum_{i \in N} u_i(a) = \sum_{i \in N} u_i(a')$ for all $a, a' \in A$. For any discount factor $\delta \in (0,1)$ and any two-player zero-sum game, compute the set of all payoff vectors that can occur in an SPE of the repeated game $RG(\delta)$.

Question 3

Consider the three-player coordination game shown below.

<table>
<thead>
<tr>
<th></th>
<th>$A$</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>1,1,1</td>
<td>0,0,0</td>
</tr>
<tr>
<td>$B$</td>
<td>0,0,0</td>
<td>0,0,0</td>
</tr>
</tbody>
</table>

Show that each player’s minmax payoff is 0, but that there is $\varepsilon > 0$ such that in every SPE of the repeated game $RG(\delta)$, regardless of the discount factor $\delta$, every player’s payoff is at least $\varepsilon$. Why does this example not violate the Fudenberg-Maskin folk theorem?

Question 4

Consider a repeated game with imperfect public monitoring. Assume that the action space and signal space are finite. Let $E(\delta)$ be the set of expected payoff vectors that can be achieved in perfect public equilibrium, where public randomization is available each period. Show that if $\delta < \delta'$, then $E(\delta) \subseteq E(\delta')$.

Question 5

Consider a two-player, infinitely repeated game in which players maximize average discounted value of stage payoffs with discount factor $\delta \in (0,1)$. At each time $t$, simultaneously
each player $i$ invests $x_{i,t} \in \{0, 1\}$ in a public good, $y_t \in \{0, 1\}$, where

$$
\mathbb{P}(y_t = 1|x_{1,t}, x_{2,t}) = \begin{cases} 
2/3 & \text{if } x_{1,t} + x_{2,t} = 2 \\
1/2 & \text{if } x_{1,t} + x_{2,t} = 1 \\
r & \text{if } x_{1,t} + x_{2,t} = 0
\end{cases}
$$

where $r \in (1/3, 5/12)$ is a parameter. The stage payoff of player $i$ is $4y_t - x_{i,t}$.

(1) Assuming that all the previous moves are publicly observable, compute the most efficient symmetric subgame-perfect equilibrium (for each $\delta \in (0, 1)$).

(2) Assume the previous levels of public goods (i.e., $y_s$ with $s < t$) are publicly observable but individual investments are not. Find the range of $\delta$ under which the grim trigger strategy profile is a public perfect equilibrium (Grim trigger: $x_{1,t} = x_{2,t} = 0$ if $y$ has ever been 0 and $x_{1,t} = x_{2,t} = 1$ otherwise).

(3) In part (b), find the range of $\delta$ under which the following is a public perfect equilibrium: start with $x_{1,t} = x_{2,t} = 1$, and for any $t > 0$, select $x_{1,t} = x_{2,t} = y_{t-1}$.