14.127 Behavioral Economics
(Lecture 2)

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0.1 Cumulative PT

- Remind from last lecture: for continuous gambles with distribution $f(x)$
  EU gives:
  \[ V = \int_{-\infty}^{+\infty} u(x) f(x) \, dx, \]
  PT gives:
  \[
  V = \int_{0}^{+\infty} u(x) f(x) \pi'(P(g \geq x)) \, dx \\
  + \int_{-\infty}^{0} u(x) f(x) \pi'(P(g \leq x)) \, dx
  \]
• Alternatively, we can write it as Riemann-Stieltjes integral

\[ V = -\int_{0}^{+\infty} u(x) \, d\pi \left(1 - P(g < x)\right) \]
\[ + \int_{-\infty}^{0} u(x) \, d\pi \left(P(g \leq x)\right) \]

• This simplifies to PT for two outcome gambles. Indeed, it is self-evident in the Riemann-Stieltjes form.
1 The endowment effect – a consequence of PT

  - Half of the subjects receives an MIT apple, and the other half receives $10.
  - Then willingness to pay WTP for the apple is elicited from subjects with money, and willingness to accept WTA is elicited from subjects with mugs.

- In EU we have $WTP = WTA$ (modulo wealth effects, which are small)
• In simplified (linear) PT value getting an apple and lose $x$ is

\[ V = u(\text{apple}) + u(-x) = A - \lambda x \]

(note—there are mental accounting ideas plugged in here that is we process apple and money on separate accounts).

• Thus, in PT, one accepts when

\[ A - \lambda x \geq 0 \]

so that

\[ WTA = \frac{A}{\lambda}. \]

• In simplified (linear) PT value losing an apple and gaining $x$ is

\[ V = u(-\text{apple}) + u(x) = -\lambda A + x \]
(note, once more time we process apple and money on separate accounts).

- Thus, in PT, one pays when

$$-\lambda A + x \geq 0$$

so that

$$WTP = \lambda A.$$  

- Thus, PT gives stability to humane life, a status quo bias.
1.1 Endowment effect experiment with mugs

- Classroom of one hundred. Fifty get the mug, fifty get $20.

- One does a call auction in which people can trade mugs.

- Trading volume — “rational” expectation would be that the average trading volume should be \( \frac{1}{2} \times 50 = 25 \). Everybody has a valuation, and probability that someone with valuation higher than the market price is \( \frac{1}{2} \).

- If WTP < WTA then the trading volume is lower than \( \frac{1}{2} \).

- In experiments, the trading volume is about \( \frac{1}{4} \).
1.2 Open questions with PT

1.2.1 Open question 1: Narrow framing

- $N$ independent gambles: $g_1, ..., g_N$

- For each $i$ do you accept $g_i$ or not?

- In EU call $a_i = 1$ if accept $g_i$ and $a_i = 0$ otherwise. Your total wealth is

\[ W_0 + a_1g_1 + ... + a_Ng_N \]

and you maximize

\[ \max_{a_1, ..., a_N} Eu (W_0 + a_1g_1 + ... + a_Ng_N) . \]
• In PT we have at least two possibilities

  – Separation: \( a_i = 1 \) iff \( V_{PT}(g_i) \).

  – Integrative: solve \( \max_{a_1, \ldots, a_N} V_{PT}(a_1 g_1 + \ldots + a_N g_N) \).

• Separation is more popular, but unlikely in for example in stock market, or venture capital work.

• KT don’t tell whether integration or separation will be chosen. That is one of the reasons PT has not been used much in micro or macro.

• How to fix this problem?
– Integration as far as possible subject to computational costs.

– Natural horizon between now and when I need to retire.

– Do what makes me happier, \( \max(\text{separation, integration}) \). That would be an appealing general way to solve the problem.

* Problem, each everyday gamble is small against the background of all other gambles of life.

* So, an EU maximizer would be locally risk neutral.

* And also a PT maximizer would be locally risk neutral whenever he or she accepts integrationist frame.
1.2.2 Horizon problem — a particular case of the framing problem

- Stock market.

  - Yearly values

    standard deviation $\sigma T^{\frac{1}{2}} = 20\%$ per year where $T \sim 250\text{days}$,

    mean $\mu T = 6\%$ per year.

  - Daily values

    $\sigma = \frac{20\%}{250^{\frac{1}{2}}} \quad \mu = \frac{6\%}{T}$
– Assume that a PT agent follows the rule: “accept if $\frac{\text{Risk premium}}{\text{St. dev.}} > k$” (PS1 asks to show existence of such an PT agent).

– So, a PT agent with yearly horizon invests if

$$\frac{6\%}{20\%} > k^*$$

– A PT agent with daily horizon invests if

$$\frac{\mu}{\sigma} = \frac{.024}{1.3} \approx .01 \ll k^*$$

– This is not even a debated issue, because people don’t even know how to start that discussion

– Kahneman says in his Nobel lecture that people use “accessible” horizons.
E.g. in stock market 1 year is very accessible, because mutual funds and others use it in their prospectuses.

Other alternatives – time to retirement or time to a big purchase. or “TV every day”.

In practice, for example Barberis, Huang, and Santos QJE 2001 postulate an exogenous horizon.
1.2.3 Open question 2: Risk seeking

- Take stock market with return $R = \mu + \sigma n$ with $n \sim N(0, 1)$.

- Invest proportion $\theta$ in stock and $1 - \theta$ in a riskless bond with return 0.

- Total return is
  \[ \theta R + (1 - \theta) 0 = \theta (\mu + \sigma n). \]

- Let’s use PT with $\pi(p) = p$. The PT value is
  \[ V = \int_{-\infty}^{+\infty} u(\theta (\mu + \sigma n)) f(n) \, dn \]
• Set \( u(x) = x^\alpha \) for positive \( x \) and \( -\lambda |x|^\alpha \) for negative \( x \).

• Using homothecity of \( u \) we get

\[
V = \int_{-\infty}^{+\infty} |\theta|^\alpha u(\mu + \sigma n) f(n) \, dn \\
= |\theta|^\alpha \int_{-\infty}^{+\infty} u(\mu + \sigma n) f(n) \, dn
\]

• Thus optimal \( \theta \) to equals 0 or \( +\infty \) depending on sign of the last integral.

• Why this problem? It comes because we don’t have concave objective function. Without concavity it is easy to have those bang-bang solutions.
One solution to this problem is that people maximize $V^{EU} + V^{PT}$. 
1.2.4 Open question 3: Reference point

- Implicitly we take the reference point to be wealth at $t = 0$. Gamble is $W_0 + g$ and

$$V^{PT} = V^{PT} (W_0 + g - R)$$

- But how $R_t$ evolves in time?

- In practice, Barberis, Huang, and Santos QJE 2001 (the most courageous paper) postulate some ad hoc exogenous process. People gave them the benefit of a doubt.
1.2.5 Open question 4: Dynamic inconsistency

• Take a stock over a year horizon. Invest 70% on Jan 1st, 2001.

• It’s Dec 1, 2001. Should I stay invested?

• If the new horizon is now one month, I may prefer to disinvest, even though on Jan 1, 2001, I wanted to keep for the entire year.

• By backward induction, Jan 1 guy should disinvest!
1.2.6 Open question 5: Doing welfare is hard

• Why? Because it depends on the frame.

• Take $T = 250$ days of stock returns $g_i \sim N(\mu, \sigma^2)$. Integrated $V^{PT}(\sum g_i) = V^I$ and separated $V^{PT} = V^S$.

• The cost of the business cycle (Lucas). Suppose $c =$average monthly consumption. Assume simple consumption shocks:

$$c_t = c + \varepsilon_t$$

with normal iid $\varepsilon_t$.

• What is PT reference point? Take $R_t = c = 0$. 
• With PT integrated over one year

\[ V^{PT} \left( \sum \varepsilon_t \right) = V^{PT} \left( 12^{\frac{1}{2}} \sigma \varepsilon n \right) = \left( 12^{\frac{1}{2}} \sigma \varepsilon \right)^\alpha V^{PT} (n) < 0. \]

• With segregated PT

\[ V^{PT} = 12 \sigma^\alpha V^{PT} (n) \]

• Which frame is better?
1.3 Next time

- Lucas calculation of costs of business cycle. In practice people care about business cycles, and election are decided on those counts.

- Problem Set — next time. One question — try to circumvent one of the problems.

- Readings on heuristics and biases, the Science 74 KT article and Camerer’s paper from the syllabus.