14.127 Lecture 5

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0.1 Welfare and noise. A compliment

- Two firms produce roughly identical goods

- Demand of firm 1 is

\[
D_1 = P \left( q - p_1 + \sigma \varepsilon_1 > q - p_2 + \sigma \varepsilon_2 \right)
\]

where \( \varepsilon_1, \varepsilon_2 \) are iid \( N(0,1) \).

- Thus

\[
D_1 = P \left( p_2 - p_1 > \sigma (\varepsilon_1 - \varepsilon_2) \right) = P \left( p_2 - p_1 > \sigma \sqrt{2} \eta \right)
\]

\[
= P \left( \frac{p_2 - p_1}{\sigma \sqrt{2}} > \eta \right) = \bar{\Phi} \left( \frac{p_2 - p_1}{\sigma \sqrt{2}} \right)
\]

where \( \eta \) is \( N(0,1) \) and \( \bar{\Phi} = 1 - \Phi \), with \( \Phi \) cdf of \( N(0,1) \).
Unlike in $\varepsilon \equiv 0$ case, here the demand is not dramatically elastic.

Slope of demand at the symmetric equilibrium $p_1 = p_2$

$$-rac{\partial}{\partial p_1} D_1 = -\frac{\partial}{\partial p_1} \Phi \left( \frac{p_2 - p_1}{\sigma \sqrt{2}} \right) = \phi \left( \frac{p_2 - p_1}{\sigma \sqrt{2}} \right) \frac{1}{\sigma \sqrt{2}}$$

$$= \phi (0) \frac{1}{\sigma \sqrt{2}}$$

and “modified” elasticity

$$\eta = -\frac{1}{D_1 \frac{\partial}{\partial p_1} D_1} = \phi (0) \frac{1}{\sigma \sqrt{2}} = \frac{\sqrt{\pi}}{\sigma}$$

because $D_1 = \frac{1}{2}$.

When $\sigma \to 0$ then $\eta \to \infty$. Even though the “true” elasticity is $\infty$ the measured elasticity is lower $\eta < \eta^{\text{true}}$. 
• Open question: how to correct that bias?
0.2 How to measure the quantity of noise $\sigma$?

- Give people $n$ mutual funds and ask them to pick their preferred and next preferred fund.

- Assume that all those funds have the same value $q_A = q_B$

- People do

$$\max q_i - p_i + \sigma \varepsilon_i = s_i$$

- Call $A$ - the best fund, $B$ - the second best fund, $s_A \geq s_B \geq$ all other funds.

- Increase $p_A$ by $\Delta p$. At some point the consumer is indifferent between $A$ and $B$.

$$q_A - p_A + \sigma \varepsilon_A - \Delta p = q_B - p_B + \sigma \varepsilon_B$$
- If $p_A = p_B$ then
  \[ \Delta p = \sigma (\varepsilon_A - \varepsilon_B) \]

  or
  \[ \Delta p = \sigma (\varepsilon_{1:n} - \varepsilon_{2:n}) \]

  - **Proposition.** For large $n$
  \[ \Delta p = B_n \sigma \]

    where $B_n$ is the parameter of Gumbel attraction, 

    \[ B_n = \frac{1}{n f \left( \frac{1}{n} \right) \bar{F}} \]
0.3  Could the fees be due to search costs?

  - Suppose you have $x = 200,000 and you keep it for 10 years.
  - You pay 1.5%/year and thus lose $200,000 \times 1.5\% = 3,000$ a year.

- Competing explanation – people don’t know that two index mutual fund are the same thing.
0.4 Open questions

- What are the regulatory implications of consumer confusion?

- Where does confusion $\sigma \varepsilon_i$ comes from? For instance, provide a cognitive model that gives a microfoundation for this “noise”

- Find a model that predicts the level of the confusion $\sigma$? e.g., in the mutual fund market, give a model that predicts the reasonable order of magnitude.

- Find a model that predicts how $\sigma$ varies with experience?

- How do firms increase/create confusion $\sigma$?
• Empirically, how could we distinguish whether profits come from true product differentiation, search costs, or confusion noise?

• Devise a novel empirical strategy to measure an effect related to the material of lectures 3 to 5.
0.5 Competition and confusion

- **Proposition.** Firms have an incentive to increase the confusion. The effect is stronger, the stronger is competition.

- Example – cell phone pricing.

- Symmetry of firms is important here. If there is a firm that is much better than others, then it wants to have very low $\sigma$ to signal this.
• Proof.

- Consider $n$ identical firms and symmetric equilibria.

\[
D_1 = P \left( q - p_1 + \sigma_1 \varepsilon_1 + V (\sigma_1) > \max q - p_i + \sigma_i \varepsilon_i + V (\sigma_i) \right)
\]
\[
= P \left( q - p_1 + \sigma_1 \varepsilon_1 + V (\sigma_1) > \max q - p^* + \sigma^* \varepsilon_i V (\sigma^*) \right)
\]

where $V (\sigma_i)$ is the utility of complexity $\sigma_i$ (equated with confusion).

- Denote $M_{n-1} = \max_{i=2,\ldots,n} \varepsilon_i$. In equilibrium

\[
D_1 = P \left( p^* - p_1 + \sigma_1 \varepsilon_1 + V (\sigma_1) - V (\sigma^*) > \sigma^* M_{n-1} \right)
\]
\[
= P \left( \varepsilon_1 > \frac{\sigma^* M_{n-1} - p^* - p_1 + V (\sigma_1) - V (\sigma^*)}{\sigma_1} \right) = \tilde{E} \tilde{F}(c)
\]

- At the equilibrium, $p_1 = p^*$, $\sigma_1 = \sigma^*$, and by symmetry

\[
D_1 = \frac{1}{n}
\]
Let us check it to develop flexibility with tricks of the trade. First note that

\[ P(M_{n-1} < x) = P((\forall i) \epsilon_i < x) = F(x)^{n-1} \]

* Density \( g_{n-1}(x) = G'_{n-1}(x) = (n - 1) F^{n-2}(x) f(x) \).  

* Now

\[
1 - D_1 = E \left( 1 - \bar{F}(M_{n-1}) \right) = E(F(M_{n-1})) = \int F(x) g_{n-1}(x) \, dx
\]

\[
= \int F(x)(n - 1) F^{n-2}(x) f(x) \, dx = \int (n - 1) F^{n-1}(x) f(x) \, dx
\]

\[
= (n - 1) \left[ \frac{F^n(x)}{n} \right]^{+\infty}_{-\infty} = \frac{n - 1}{n} = 1 - \frac{1}{n}
\]

* Thus \( D_1 = \frac{1}{n} \).
- Heuristic remark. $E \left( \bar{F}(M_{n-1}) \right) = \frac{1}{n}$. Hence $M_n = A_n + B_n \eta$, where $A_n >> B_n$ are Gumbel attraction constants. Thus $M_n \simeq A_n$. So,

$$M_n \simeq \bar{F} \left( \frac{1}{n} \right) \simeq A_n$$
- The profit \( \pi_1 = \max_{p_1, \sigma_1} E \bar{F} \left( \frac{\sigma^* M_{n-1}}{\sigma_1} - \frac{p^* - p_1 + V(\sigma_1) - V(\sigma^*)}{\sigma_1} \right) (p_1 - c_1) \)

- From FOC and envelope theorem

\[
0 = \frac{d}{d\sigma_1} \pi_1 = (p_1 - c_1) \frac{\partial}{\partial \sigma_1} D_1
\]

* Note that

\[
\frac{\partial}{\partial \sigma_1} D_1 = E \left( -f(c_n) \left( -\frac{\sigma^* M_{n-1}}{\sigma_1^2} - \frac{-(p^* - p_1 + V(\sigma_1) - V(\sigma^*))}{\sigma_1^2} \right) \right)
\]

* In equilibrium, \( c_n = M_{n-1} \), hence

\[
0 = \frac{\partial}{\partial \sigma_1} D_1 = E \left( -f(M_{n-1}) \left( -\frac{M_{n-1} \sigma^*}{\sigma^*} - \frac{V'(\sigma_1)}{\sigma^*} \right) \right)
\]

\[
= E \left( f(M_{n-1}) \frac{M_{n-1}}{\sigma^*} \right) + E \left( f(M_{n-1}) \frac{V'(\sigma_1)}{\sigma^*} \right)
\]
Hence
\[ V' \left( \sigma_1 \right) = -\frac{E \left( f \left( M_{n-1} \right) M_{n-1} \right)}{Ef \left( M_{n-1} \right)} \equiv -d_n \]

Consider some simple cases

- Uniform distribution \( d_n = 1 - \frac{2}{n} \)
- Gumbel \( d_n = \ln n + A \)
- Gaussian \( d_n \sim \sqrt{\ln n} \)

In those cases, \( V' \left( \sigma_1 \right) < 0 \).

Thus we have excess complexity.
• What happens as competition grows while $n \to \infty$?
  
  – Take the utility of noise to be $V(\sigma) = 1 - \frac{1}{2\chi} (\sigma - \sigma^{**})^2$.
  
  – Then $V'(\sigma) = \frac{-1}{\chi} (\sigma - \sigma^{**}) = -d_n$, and consequently
    
    $$\sigma = \sigma^{**} + \chi d_n$$
  
• Hence, if competition grows, the problem gets exarcerbated.
0.5.1 Open question. The market for advice works very badly. Why?

- The fund manager wants to sell their own funds.

  - Advisor charges you 1% per year for advice: he gives you stories each month that suggest some kind of trade. Otherwise, he could lose client.
1 Marketing - Introduction

- Why high prices of add-ons and low prices of printers or cars?

- Often the high add-ons fees are paid by the poor not rich who might be argued have low marginal value of money, e.g. use of credit card to facilitate transactions.

- Many goods have “shrouded attributes” that some people don’t anticipate when deciding on a purchase.
• Consider buying a printer.
  – Some consumers only look at printer prices.
  – They don’t look up the cost of cartridges.

• Shrouded add-ons will have large mark-ups.
  – Even in competitive markets.
  – Even when demand is price-elastic.
  – Even when advertising is free.
2 Shrouded attributes

- Consider a bank that sells two kinds of services.
- For price $p$ a consumer can open an account.
- If consumer violates minimum she pays fee $\hat{p}$.
- WLOG assume that the true cost to the bank is zero.
- Consumer benefits $V$ from violating the minimum.
- Consumer alternatively may reduce expenditure to generate liquidity $V$.

<table>
<thead>
<tr>
<th>Spend normally</th>
<th>Do not violate minimum</th>
<th>Violate minimum</th>
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</thead>
<tbody>
<tr>
<td>Spend less</td>
<td>$V - e &gt; 0$</td>
<td>$V - e - \hat{p}$</td>
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</table>
2.1 Sophisticated consumer

- Sophisticates anticipate the fee $\hat{p}$.

- They choose to spend less, with payoff $V - e$

- ...or to violate the minimum, with payoff $V - \hat{p}$
2.2 Naive consumer

- Naive consumers do not fully anticipate the fee $\hat{p}$.

- Naive consumers may completely overlook the aftermarket or they may mistakenly believe that $\hat{p} < e$.

- Naive consumers will not spend at a reduced rate.

- Naive consumer must choose between foregoing payoff $V$ or paying fee $\hat{p}$.
2.3 Summary of the model

- Sophisticates will buy the add-on iff $V - \hat{p} \geq V - e$.

- Naives will buy the add-on iff $V - \hat{p} \geq 0$.

- $D(x_i)$ is the probability that a consumer opens an account at bank $i$.

- For sophisticated consumer

$$D_i = P \left( q - p_i + \max (V - e, V - \hat{p}_i) + \sigma \varepsilon_i > q - p^* + \max (V - e, V - \hat{p}) \right)$$

$$= P \left( \sigma \varepsilon_i + x > \sigma \max_{j \neq i} \varepsilon_j \right)$$
• Let $\alpha$ – fraction of rational (sophisticated) consumers, $1 - \alpha$ – fraction of irrational (naive) consumers

• Profit earned from rational consumers

$$
\pi = \alpha \left( p + \hat{p}1_{\hat{p} \leq e} \right) D (-p + \max (V - e, V - \hat{p}) + p^* - \max (V - e, V - p^*)
$$

• Profit earned on irrational consumers

$$(1 - \alpha) \left( p + \hat{p}1_{\hat{p} \leq V} \right) D (-p + p^*)$$
Proposition. Call $\alpha^\dagger = 1 - \frac{e}{V}$ and $\mu = \frac{D(0)}{D'(0)}$.

- If $\alpha < \alpha^\dagger$, equilibrium prices are
  \[ p = -(1 - \alpha) V + \mu \]
  \[ \hat{p} = V \]
  and only naive agents consume the add-on.

- If $\alpha \geq \alpha^\dagger$, prices are
  \[ p = -e + \mu \]
  \[ \hat{p} = e \]
  and all agents consume the add-on.
Corollary. If $\alpha < \alpha^\dagger$, then the equilibrium profits equal

$$\pi = \alpha p D (0) + (1 - \alpha) (p + \hat{p}) D (0)$$

$$= (p + (1 - \alpha) \hat{p}) D (0) = \mu D (0) = \frac{\mu}{n}$$

- Firms set high mark-ups in the add-on market.

- If there aren’t many sophisticates, the add-on mark-ups will be inefficiently high: $\hat{p} = V > e$. 
• High mark-ups for the add-on are offset by low or negative mark-ups on the base good.

• To see this, assume market is competitive, so \( \mu \simeq 0 \).

  – Loss leader base good: \( p^* \approx -(1 - \alpha) V < 0 \).

• Examples: printers, hotels, banks, credit card teaser, mortgage teaser, cell phone, etc...
• The shrouded market becomes the profit-center because at least some consumers don’t anticipate the shrouded add-on market and won’t respond to a price cut in the shrouded market.

• Interpretations

  – bounded rationality, people don’t see small print.

  – overconfidence – people believe they will not fail prey to small print penalties.