14.127 Lecture 7

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1 Learning in games

- Drew Fudenberg and David Levine, The Theory of Learning in Games
1.1 Fictitious play

- Let $\gamma_t^i$ denotes frequencies of $i$’s opponents play

$$\gamma_t^i(s_{-i}) = \frac{\text{number of times } s_{-i} \text{ was played till now}}{t}$$

- Player $i$ plays the best response $BR(\gamma_t^i)$

- Big concerns:
  - Asymptotic behavior: do we converge or do we cycle?
  - If we converge, then to what subset of Nash equilibria?

- Caveat. Empirical distribution need not converge
1.2 Replicator dynamics

- Call $\theta^i_t(s^i) = \text{fraction of players of type } i \text{ who play } s_i$.

- Postulate dynamics
  
  - In discrete time
    
    $$\vec{\theta}^i_{t+1} = (\theta^i_{t+1}(s_1), \ldots, \theta^i_{t+1}(s_n)) = \vec{\theta}^i_t + \lambda \left( BR(\vec{\theta}^{-i}_t) - \vec{\theta}^i_t \right)$$
  
  - In continuous time
    
    $$\frac{d}{dt} \vec{\theta}^i_{t+1} = \lambda \left( BR(\vec{\theta}^{-i}_t) - \vec{\theta}^i_t \right)$$

- Then analyze the dynamics: chaos, cycles, fixed points
1.3 Experience weighted attraction model, EWA

- Camerer-Ho, Econometrica 1999

- Denote $N_t =$number of “observation equivalent” past responses such that

$$N_{t+1} = \rho N_t + 1$$

- Denote

  - $s_{ij}$ — strategy $j$ of player $i$
  
  - $s_i(t)$ — strategy that $i$ played at $t$
  
  - $\pi_i(s_{ij}, s_{-i}(t))$ — payoff of $i$
• Perceived payoff with parameter $\phi \in [0, 1]$

$$A_{ij}(t) = \frac{1}{N_t} \left[ \phi N_{t-1} A_{ij}(t-1) + \left( \delta + (1 - \delta) 1_{s_{ij}=s_i(t)} \right) \pi_i (s_{ij}, s_{-i}(t)) \right]$$

• Attraction to strategy $j$

$$\rho_{ij}(t) = \frac{e^{\lambda A_{ij}(t)}}{\sum_{j'} e^{\lambda A_{ij'}(t)}}$$

• At time $t + 1$ player $i$ plays $j$ with probability $\rho_{ij}(t)$

• Free parameters: $\delta, \phi, \rho, A_{ij}(0), N(0)$
• Some cases
  
  – If $\delta = 0$ – reinforcement learning (called also law of effect). You only reinforce strategies that you actually played

  – If $\delta > 0$ – law of simulated effect

  – If $\phi = 0$ – agent very forgetful

• Proposition. If $\phi = \rho$ and $\delta = 1$ then EWA is a belief-based model. Makes predictions of fictitious play.

• If $N(0) = \infty$ and $A_{ij}(0) =$ equilibrium payoffs then EWA agent is a dogmatic game theorist.
1.3.1 Functional EWA (f-EWA)

- Has just one parameters. Other endogenized. But still looks like data fitting.

- Camerer, Ho, and Chong working paper

- They look after parameters that fit all the games

- They $R^2$ is good

- Other people in this field: Costa-Gomez, Crawford, Erev
1.3.2 Critique

- Those things are more endogenous than postulated.

- E.g. fictitious play guy does not detect trends, but people do detect trends.

- How do you model patterns, how do you detect patterns. Whole field of pattern recognition in cognitive psychology.

- If you are interested in strategy number 1069, then strategy 1068 should benefit also. There is some smoothing.
1.4 Cognitive hierarchy model of one-shot games

- Camerer - Ho, QJE forthcoming

- $s_i^j$ – strategy $j$ of player $i$ and $\pi_i(s_i, s_{-i})$ – profit of player $i$

- Each level 0 player:
  - just postulates that other players play at random with probability $\frac{1}{N}$
  - best responses to that belief
• Each level $k$ player:

  - thinks that there is a fraction of players of levels $h \in \{0, \ldots, k - 1\}$
  
  - proportions are $g_k(h) = \frac{f(h)}{\sum_{h'=0}^{k-1} f(h')}$ and $g_k(h) = 0$ for $h \geq k$

  - $k$-players best response to this belief

• Camerer-Ho postulate a Poisson distribution for $f$ with parameter $\tau$,

  $$f(k) = e^{-\tau} \frac{\tau^k}{k!}$$

  with $Ek = \sum_{k \geq 0} k f(k) = \tau$.

• The authors calibrate to empirical data and find the average $\tau \simeq 1.5$. 
1.5 An open problem – asymmetric information

- James has a plant with value $V$ uniformly distributed over $[0, 100]$.

- James knows $V$, you don’t

- You are a better manager than James; the value to you is $\frac{3}{2}V$

- You can make a take it or leave it offer to James of $x$.

- What you would do?
• Empirically people offer between 50 and 75. But that is not the rational value.

• **Proposition.** The rational offer is 0.

• **Proof.** You offer $x$.
  
  - If $V > x$ then James refuses, and your payoff $W = 0$.
  
  - If $V \leq x$ then $V$ is uniformly distributed between 0 and $x$. Hence your expected value is $W = \frac{3}{2} \cdot \frac{x}{2} - x = -\frac{x}{4}$.
  
  - Hence best you can do is set $x = 0$. QED
1.5.1 How to model people’s choice?

• This game is not covered by cognitive hierarchy model. It is a single person decision problem.

• Maybe people approximate $V$ by, for example, a unit mass at the mean $V = 50$?

• Other question. You own newspaper stand. You can buy newspaper for $1 and have a chance to sell for $4. There are no returns. The demand is uniform between 50 and 150. How many would you buy?

• Something along those lines will be in Problem Set 3.