14.127 Behavioral Economics. Lecture 11
Introduction to Behavioral Finance

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2 Finance

Andrei Schleifer, Efficient Markets (book)

2.1 Closed end funds

- Fixed number of shares traded in the market
- The only way to walk away is to sell fund’s share
• NAV – Net Asset Value – is the dollar value of single share computed as the value of the assets inside the shell net of liabilities divided by the number of the shares

• Discount $= \frac{(\text{NAV}-\text{Share Price})}{\text{NAV}}$

• The discount substantially decreased in early 80s.
2.2 Simplest limited arbitrage model

- One risky asset $Q$ and one riskless asset (interest rate $r$)

- Two periods,
  - $t = 0$ trading,
  - $t = 1$ dividend $D = \bar{D} + \sigma z$ where $z \sim N(0, 1)$

- CARA expected utility $U(x) = -e^{-\gamma x}$.

- Buy $q$ of the stock at price $p$ and put $W - q$ in bonds.
• Payoff \( x = qD + (1 + r)(W - pq) \) and
\[
EU = -Ee^{-\gamma(qD + (1+r)(W-pq))}
\]

• Use \( Ee^{a+bz} = e^{a+\frac{b^2}{2}} \) and get
\[
EU = -e^{-\gamma(q\bar{D}+(1+r)(W-pq))}-\gamma\frac{q^2\sigma^2}{2}
\]

• Thus the agent maximizes \( \max_q \gamma(q\bar{D} + (1 + r)(W - pq)) + \frac{\gamma^2q^2\sigma^2}{2} \) and
\[
q = \frac{\bar{D} - p(1 + r)}{\gamma\sigma^2}
\]

• This gives a downward sloping demand for stocks
Imagine there are two types of agents

- irrational buy $q^I$ of stock
- rational do maximization and buy $q^R$

In equilibrium $q^I + q^R = Q$ and $p = \frac{D - \gamma \sigma^2 (Q - q^I)}{1+r}$

Thus the price moves up with the number of irrational guys.

This falsifies the claims that arbitragers will arbitrage influence of irrational guys away.

Risky stock reacts a lot to animal spirits
2.3 Noise trader risk in financial markets

- DeLong, Schleifer, Summers, Waldmann, JPE 1990

- Two types: noise traders (naives) and arbitragers (rational) with utility
  \[ U = e^{-2\gamma W} \]

- Overlapping generations model

- NT have animal spirit shocks \( \rho_t = E\rho_{t+1}, \rho_t \sim N(\rho^*, \sigma^2_\rho) \)

- The stock gives dividend \( r \) at every period

- Call \( \lambda^i_t = \) quantity of stock held by type \( i \in \{NT, A\} \)
- Demand

\[
\max_{\lambda^i} E^i e^{-2\gamma(\lambda^i(p_{t+1} - (1+r)p_t) + (1+r)W)}
\]

- For arbitragers

\[
\lambda^A = \frac{r + E_t p_{t+1} - (1 + r)p_t}{2\gamma\sigma_{t+1}^2}
\]

- We postulate \(E^{NT}p_{t+1} = Ep_{t+1} + \rho_t\) and

\[
\lambda^{NT} = \lambda^A + \frac{\rho_t}{2\gamma\sigma_{t+1}^2}
\]
• Call $\mu$ – the fraction of noise traders, supply of stock is 1

• In general equilibrium

\[
(1 - \mu) \lambda_t^A + \mu \lambda_{t}^{NT} = 1
\]

• Thus

\[
\lambda_t^A + \frac{\mu \rho_t}{2\gamma \sigma_{t+1}^2} = 1
\]

• Solving for price $p_t$

\[
p_t = \frac{1}{1 + r} \left( r + E_t p_{t+1} - 2\gamma \sigma_{t+1}^2 + \mu \rho_t \right)
\]
• Solving recursively

\[
E_{t-1}p_t = \frac{r}{1 + r} + \frac{E_{t-1}p_{t+1} - 2\gamma \sigma_{t+1}^2 + \mu \rho^*}{1 + r}.
\]

• In stationary equilibrium

\[
E_{t-1}p_t = \frac{1}{r} \left( r - 2\gamma \sigma_{t+1}^2 + \mu \rho^* \right)
\]

• Also,

\[
\sigma_{t+1}^2 = \text{Var} \left( \frac{1}{1 + r} \mu \rho_t \right) = \frac{\mu^2}{(1 + r)^2} \sigma_\rho^2
\]
• Plugging in

\[ p_t = 1 + \frac{\mu (\rho_t - \rho^*)}{1 + r} + \frac{\mu \rho^*}{r} - 2\gamma \frac{\mu^2}{(1 + r)^2} \sigma^2 \]

with the second term reflecting bullish/bearish behavior, the third term reflecting average bullishness of NT, and the forth term reflecting riskiness of stock due to changes in animal spirits.

• Even a stock with riskless fundamentals is risky because of the presence of NT, and thus the price cannot be arbitrated away.
2.3.1 Problems:

- Price can be negative

- Deeper. Remind if there is no free lunch then we can write

\[ p_t = E \left[ \frac{M_{t+1}}{1 + r} (p_{t+1} + D_{t+1}) \right] \]

for a stochastic discount factor \( M_{t+1} \).

- Iterating

\[ p_t = E \left[ \frac{M_{t+1}}{1 + r} \left( E \left[ \frac{M_{t+2}}{1 + r} (p_{t+2} + D_{t+1}) \right] + D_{t+1} \right) \right] \]
- In general

\[ p_t = \sum_{i=1}^{k} \frac{r}{(1 + r)^i} + \frac{1}{(1 + r)^k} E[M_{t+1} \ldots M_{t+k}p_{t+k}] \]

\[ = 1 - \frac{1}{(1 + r)^k} + \frac{1}{(1 + r)^k} E[M_{t+1} \ldots M_{t+k}p_{t+k}] \]

- If you constrain the price to be positive, then

\[ p_t \geq 1 \]

- One reference on this, Greg Willard et al (see his Maryland website)