1 DG monopoly with Fixed Types

Buyer-Seller: R-N, \( \delta \leq 1 \); 2 periods.

Buyer: \( v_i \) per period, \( 0 < v_L < v_H \),

\( x_{it} \) is prob buyer \( i \) consumes in period \( t \).

Seller: \( c = 0 \), \( \Pr(v_H) = \beta \).

- Full-Commitment:

Menu \( (X_i, T_i)_{i=L,H} \), where \( X_i = x_{i1} + \delta x_{i2} \).

Seller: \( (1 - \beta)T_L + \beta T_H \rightarrow X_i, T_i^{\max} \), s.t.

\[
\begin{align*}
  v_i X_i - T_i & \geq 0, & i = L, H \\
  v_i X_i - T_i & \geq v_i X_j - T_j, & i, j = L, H \\
  0 & \leq x_{it} \leq 1, & i = L, H; t = 0, 1.
\end{align*}
\]

IRL, ICH are binding:

\[
(1 - \beta)v_L X_L + \beta(v_H X_H - (v_H - v_L)X_L) \rightarrow X_L, X_H^{\max}.
\]

Thus, \( X_H = 1 + \delta \equiv \Delta \). Set \( \beta^* \equiv \frac{v_L}{v_H} \).

If \( \beta < \beta^* \), \( X_L = \Delta, T_L = T_H = v_L \Delta \) (\( P = v_L \)).

Otherwise, \( X_L = 0 = T_L, T_H = v_H \Delta \).

- Selling DG: No-Commitment. (\( \beta > \beta^* \))

\( P_t \) is price in period \( t \).

If object is sold in period \( t \), it is consumed in each period thereafter.

Let \( \beta_t = \Pr(i = H | t) \), \( \beta_1 = \beta \), \( \beta_2 = \beta_2(I_1) \), where \( I_1 \)

is the outcome (information set) of period 1.
Period 2 (as before) depends on $\beta_2 \geq \beta^*$.  

Period 1: $L$ gets zero surplus, accepts $P_1 \leq v_L\Delta$.  

Type $H$ decision depends on $Exp$ of $t = 2$:  

$P_2 = v_H \rightarrow H$ accepts $P_1 \leq v_H\Delta$.  

$P_2 = v_L \rightarrow H$ accepts $P_1 \leq v_H + \delta v_L \equiv P^*$.  

Seller’s options: (1) $P_1 = ER = v_L\Delta$.  

(2) $P_2 = v_L$, $P_1 = P^*$,  

$ER = (1 - \beta)\delta v_L + \beta P^* = \beta v_H + \delta v_L (> ER^{(1)})$.  

(3) (mixed str) Seller rnds over $P_2$, $\sigma = \Pr(P_2 = v_H)$; buyer $H$ rnds over buying in $t = 1$ ($\gamma$ is prob).  

Seller indiff: $v_L = \beta_2 v_H$, thus  

$$\beta_2 = \beta^* = \frac{\beta(1 - \gamma)}{\beta(1 - \gamma) + (1 - \beta)}; \quad \gamma = \frac{\beta - \beta^*}{\beta(1 - \beta^*)}.$$  

Buyer indiff:  

$$v_H\Delta - P_1 = \delta (1 - \sigma)(v_H - v_L); \quad \sigma = 1 - \frac{v_H\Delta - P_1}{\delta(v_H - v_L)}.$$  

Seller’s revenue:  

$$\beta\gamma P_1 + \delta [\beta(1 - \gamma)(\sigma v_H + (1 - \sigma)v_L) + (1 - \beta)(1 - \sigma)v_L]$$  

Substitute either $P_1$ or $\sigma$. Linear objective.  

Solution: $P_1 = v_H\Delta$, $\sigma = 1$.  

$$ER = \beta v_H(\gamma\Delta + (1 - \gamma)\delta).$$
When $\beta \to \beta^*$, $\gamma \to 0$, $ER \to \delta \beta v_H$. No randomizing.

When $\beta \to 1$, $\gamma \to 1$, $ER \to \beta \Delta v_H$. Randomizing is preferred.

Note, by “randomizing” seller still sells only to a high-valued buyer, but, with no commitment, sometimes no sale happens in period 1.

- Renting without Commitment.

Buyer pays $R_t$ to consume in period $t$.

This would help if types were not fixed: with iid types seller can optimize each period, while selling still suffers competition from future selves.

(+ ) Rachet effect: cannot commit not to raise the price in period 2.

Period 2: $R_2 = v_H(= v_L)$ if $\beta_2 > (\leq) \beta^*$.

Two $\beta$’s possible (reject/accept!). Here, they are the same.

Period 1: (1) $R_1 = v_L$, $R_2 = v_H$, $ER = v_L + \delta \beta v_H$.

(2) Separating regime: $v_H - R_1 \geq \delta (v_H - v_L)$. $ER = \beta (v_H - \delta (v_H - v_L)) + \delta (\beta v_H + (1 - \beta) v_L) = \beta v_H + \delta v_L > ER^{(1)}$ (here, and from now on, $\beta_t$ is probability of $v_H$ conditional on rejection.)

(3) Semi-separating regime: $H$ rents with prob $\gamma = \frac{\beta - \beta^*}{\beta(1 - \beta^*)}$, seller is indifferent between setting $R_2$ to $v_L$ or $v_R$ after rejection.

Seller’s probability of $R_2 = v_H$ is $\sigma$.

As before: $\sigma = 1$, $R_1 = v_H$. $ER$ the same.
• More than two periods. $\beta_t$ is prob of $v_H$ conditional on rejected before.

Suppose there exists $t < T$, such that $\beta_t < \beta^*$, consider lowest possible $t$. Then, $R_\tau = v_L$ for all $\tau \geq t$.

Consider period $t - 1$. Since $\beta_{t-1} \geq \beta^*$, there are high types that pay $R_{t-1}$ and signal who they are.

To do so, $v_H - R_{t-1} \geq (v_H - v_L)\delta(1 + \delta + \cdots + \delta^{T-t})$.

If, however, $\delta(1 + \delta) > 1$, $R_{t-1} < v_L$ (cannot happen). Then, $\beta_t \geq \beta^*$ for all $t$. Not much revelation possible.

Suppose $\beta$ is close to $\beta^*$.

Selling: Separation is optimal with $T = 2$. If $T = 3$, the seller can set $P_1 = v_H + (\delta + \delta^2)v_L$, $P_2 = (1 + \delta)v_L$.

Renting when $T = 3$:

(1) Set $R_1 > v_L$, so that $\beta_2 = \beta^*$. Remaining payoff is $(1 + \delta)v_L$. In period 1, $R_1 \leq v_H$, and probability of sale is $< \beta$. Worse than selling.

(2) $R_1 = v_L$, and then two-periods full separation. Worse than selling again because, $\beta > \beta^*$.

• Renegotiation-proof contracts.

Sequential Pareto-Optimality.

$T = 2$, PO means $P_2 = v_H(= v_L)$ if $\beta_2 > (\leq) \beta^*$. Exactly the same requirement as with no-commitment.

Previous cases can be represented as renegotiation-proof contracts.