1 Adverse Selection

A bidder bids on a company owned by James. Both James and the bidder are risk neutral. The value of the company to James, \( v \), is randomly distributed between 0 and 1 with the following probability density function:

\[
f(v) = \begin{cases} 
2v & \text{if } v \in [0, 1] \\
0 & \text{otherwise}
\end{cases}
\]

James knows its true value. The bidder doesn’t know its true value but does know that the company is worth 50\% more to her than it is to James. (She can run the company more efficiently that he can.) For example, if the company is worth 1 to James, then it is worth 1.5 to the bidder. The bidder will make a take-it-or-leave-it offer to James. If her offer is above his value he’ll sell it to her. If her offer is below his value, he’ll keep the company.

1. What would you bid if you had only 2 minutes to make an offer?
2. Show that a rational bidder is indifferent to bidding any amount between 0 and 1.
3. Knowing the previous result, give the intuition of what a rational bidder will bid in the 3 following variations of the problem. Then try to prove it.
   
   (a) The bidder runs the company 25\% rather than 50\% better than James.
   (b) The pdf is \( f(v) = 3v^2 \) instead of the function give above.
   (c) The bidder is risk averse.
4. Give a real life example where you think this kind of set-up could apply.
2 Prospect Theory

Call $G$ the gamble with probability .5 win 1000, with probability .5 lose 550. We usually note $G = (1000, .5; -550, .5)$.

Consider an agent with a probability weighting function $\pi(p)$ assumed to be non-decreasing, and with the following PT value function:

$$u(x) = \begin{cases} 
  x & \text{if } x \geq 0 \\
  2x & \text{if } x \leq 0
\end{cases}$$

1. What will this agent choose among:
   
   (a) do not participate,
   
   (b) play $G$ one time,
   
   (c) play $G$ two times with a single payment done at the end by adding up the two results.

2. What will he do if he has also the extra option:

   (d) play $G$ one time, see the result and have the option of playing it a second time. A single payment is done at the end.

3. Give an example of a situation:

   (a) where people will aggregate the risks and take their decision based on the final outcome,

   (b) where they will do the opposite

4. Do you think analyzing dynamic decisions using Prospect Theory will be easy? Why?