1 Problems with happiness research

- Measurement problem. Even if the true relation between happiness was linear \( u = ky \), you will get a concave pattern if the scale of happiness is bounded and the scale of wealth is unbounded.

  - the meaning of 7 on the scale 1,2,...,10 changes.

* Similarly, if you ask people how high they are on the scale 1,2,...,10 then the average answer may not differ across years even if height goes up

- Problem with relative consumption theory

  - people don’t take steps to change locations to be ahead of others
2 Self-control problems and hyperbolic discounting

Reading: Thaler, chapter on Intertemporal Choice, *Winner’s Curse.*
2.1 Orthodox intertemporal choice

2.1.1 A three-period consumption problem

• Assume that at date 0 the individual picks consumption $c_0, c_1, c_2$ for dates 0, 1, and 2. He has wealth $W_0$ and can put savings in the bank with interest rate $r$

  – at time 1 he has wealth $W_1 = (1 + r)(W_0 - c_0)$

  – at time 2 he has wealth $W_2 = (1 + r)(W_1 - c_1)$

  – at time 2 he consumes remaining wealth $c_2 = W_2$.

  – The agent maximizes $u(c_0, c_1, c_2)$ over $c_0, c_1, c_2, W_1, W_2$ subject to the above three constraints.
• We can substitute out for $W_1, W_2$ and reduce the consumption problem to:

$$\max_{\{c_0, c_1, c_2\}} u(c_0, c_1, c_2)$$

subject to the constraint:

$$W_0 = c_0 + \frac{c_1}{(1 + r)} + \frac{c_2}{(1 + r)^2}.$$  

• In other words, the NPV of consumption is equal to the value of wealth, $W_0$, at period 0.
• For a $T + 1$ period problem the constraint would take the form

$$W_0 = c_0 + \frac{c_1}{(1 + r)} + \frac{c_2}{(1 + r)^2} + \ldots + \frac{c_T}{(1 + r)^T}.$$

• Postulate

$$u(c_0, c_1, c_2) = \Delta_0 u(c_0) + \Delta_1 u(c_1) + \Delta_2 u(c_2)$$
• To solve this problem, set up a Lagrangian:

\[
\max_{\{c_0, c_1, c_2\}} \Delta_0 u(c_0) + \Delta_1 u(c_1) + \Delta_2 u(c_2)
+ \lambda \left[ W_0 - \left( c_0 + \frac{c_1}{1 + r} + \frac{c_2}{(1 + r)^2} \right) \right]
\]

• In general when the Lagrangian maximization is \( \max L = F - \lambda G \) the first order conditions are

\[
\frac{\partial}{\partial c_i} L = 0.
\]
So, our first order condition’s are:

\[
\begin{align*}
\Delta_0 u'(c_0) &= \lambda \\
\Delta_1 Ru'(c_1) &= \lambda \\
\Delta_2 R^2 u'(c_2) &= \lambda
\end{align*}
\]

and the budget constraint is

\[
W_0 = c_0 + \frac{c_1}{(1 + r)} + \frac{c_2}{(1 + r)^2}.
\]
• To solve specify \( u(c) = \ln c \).

• Then

\[
c_t = \Delta_t (1 + r)^t \frac{1}{\lambda}
\]

• This leads to

\[
W_0 = \sum_{t=0}^{2} \frac{\Delta_t}{\lambda}
\]

or

\[
\frac{1}{\lambda} = \frac{W_0}{\sum_{t=0}^{2} \Delta_t}
\]
• Thus

\[ c_t = (1 + r)^t \frac{\Delta t}{\sum_{t=0}^{2} \Delta t} W_0 \]

• Assume that \( r = 0 \). Then

\[ c_t = \frac{\Delta t}{\sum_{t=0}^{2} \Delta t} W_0 \]
2.1.2 Time Consistency

• Imagine that the agent can reoptimize at times 1 and 2. Will he stick to time 0 determined consumption path?

• Assume $W_0 = 1$.

• At time 1 the wealth is $W_1 = (1 + r)(W_0 - c_0) = 1 - c_0 = \frac{\Delta_1 + \Delta_2}{\sum_{t=0}^{2} \Delta_t}$.

• The two period problem is analogous to the three period one we just considered with consumer consuming $c_1', c_2'$ at times 1 and 2, respectively.

• Postulate that the weights he employs $\Delta_1', \Delta_2'$ are the same as $\Delta_0, \Delta_1$, respectively (i.e. the weights depend only on distance in time from present).
• The agent consumes

\[
c'_{1} = \frac{\Delta_{0}}{\sum_{t=0}^{1} \Delta_{t}} \frac{\Delta_{1} + \Delta_{2}}{\sum_{t=0}^{2} \Delta_{t}}
\]

\[
c'_{2} = \frac{\Delta_{1}}{\sum_{t=0}^{1} \Delta_{t}} \frac{\Delta_{1} + \Delta_{2}}{\sum_{t=0}^{2} \Delta_{t}}
\]

• Hence

\[
\frac{c'_{2}}{c'_{1}} = \frac{\Delta_{1}}{\Delta_{0}}.
\]
• If those consumptions \( c'_0, c'_1 \) are the same as \( c_1, c_2 \) planned at original time 0, then

\[
\frac{c'_1}{c'_0} = \frac{c_2}{c_1} = \frac{\Delta_2}{\Delta_1}.
\]

• Hence, the condition of consistency is

\[
\frac{\Delta_1}{\Delta_0} = \frac{\Delta_2}{\Delta_1}.
\]

• In other words, there is \( \alpha \) such that

\[
\Delta_1 = \alpha \Delta_0
\]

\[
\Delta_2 = \alpha \Delta_1 = \alpha^2 \Delta_0.
\]
• Proposition. There is no time inconsistency iff there exist $\Delta_0$ and $\alpha$ such that

$$\Delta_t = \alpha^t \Delta_0,$$

i.e.

$$u = \Delta_0 \left( \ln c_0 + \alpha \ln c_1 + \alpha^2 \ln c_2 \right).$$
Proposition. If the agent, at $t = 0$, maximizes

$$V_{t=0} = \sum_{t=0}^{T} \Delta_s u(c_s)$$

and at $t = t_1$ he maximizes

$$V_{t=t_1} = \sum_{t=0}^{T-t_1} \Delta_s u(c_{t_1+s})$$

the decision problem is time consistent iff there exist $\Delta_0$ and $\alpha$ such that

$$\Delta_t = \Delta_0 \alpha^t.$$ 

Time consistent means that the optimal consumption $(c_1^*, c_2^*, ..., c_t^*)$ decided at time 0 is always optimal at any $t_1 > 0$. 