1 FAIRNESS

1.1 Ultimatum Game

- a Proposer (P) and a receiver (R) split $10

- P proposes $s$

- R can accept or reject
  - if R accepts, the payoffs are $(P,R) = (10 - s, s)$
  - if R rejects, they are $(0, 0)$

• Societies with lots of interactions
  
  – reputation is important (for example society with no or a very weak state)

  – incentives to never accept something below 50% (short term loss but long term gain)

• measure one dimension of fairness / equality
1.2 2 interesting variants

1. Market game with several proposers
   - \( n - 1 \) proposers who propose simultaneously \( s_i \)
   - 1 responder who accepts or rejects the highest offer \( s_{\text{max}} = \max s_i \)
   - empirically \( s_{\text{max}} = 10 \): incentive to offer more than the other proposers

2. Market game with several responders
   - 1 proposer
   - \( n-1 \) responders
– if all reject the offer, everybody gets 0

– if some accept, the offer is randomly assigned among the responders who accepted

• empirically $s = \varepsilon$ and it is accepted

3. It would be nice to have a model that explains all of these phenomena.
1.3 Fehr-Schmidt QJE’99

- n players

- final monetary payoffs $x_i \ i = 1\ldots n$

- utility function

$$U_i(x_1,\ldots,x_n) = x_i - \frac{\alpha_i}{n-1} \sum_j (x_j - x_i)^+ - \frac{\beta_i}{n-1} \sum_j (x_i - x_j)^+$$

where $\alpha_i \geq \beta_i \geq 0$ and $1 > \beta_i$. Notation $y^+ = \max(y,0)$

- utility of $i$ as a function of the monetary payoff of $j$ $x_j$
- if $x_j < x_i$, then $u_i = -\frac{\beta_i}{n-1}(x_i - x_j) + \text{terms independent of } x_j$

- if $x_j > x_i$, then $u_i = -\frac{\alpha_i}{n-1}(x_j - x_i) + \text{terms independent of } x_j$
• $i$ cares about the payoffs $j$ gets

• $i$ dislikes that $j$ gets more than him

• $i$ dislikes that $j$ gets less than him

• $i$ cares more about being behind than being ahead
1.4 Application to the Ultimatum Game

- player 1 is the proposer
- player 2 is the receiver
- they try to share $1
- $s = offer of the proposer
Receiver’s strategy

- if he rejects, the payoffs are 0 and $U_2 = 0$

- if he accepts
  
  - the payoffs are $x_1 = 1 - s$ and $x_2 = s$
  
  - his utility is

$$U_2 = s - \alpha_2(1 - s - s)^+ - \beta_2(s - 1 + s)^+$$

$$= \begin{cases} 
  s - \alpha_2(1 - 2s) & \text{if } \frac{1}{2} \geq s \\
  s - \beta_2(2s - 1) & \text{if } \frac{1}{2} \leq s 
\end{cases}$$

$$= \begin{cases} 
  (1 + \alpha_2)s - \alpha_2 & \text{if } \frac{1}{2} \geq s \\
  (1 - 2\beta_2)s + \beta_2 & \text{if } \frac{1}{2} \leq s 
\end{cases}$$
R accepts iff $s \in [s_2^*, 1]$, where $s_2^* = \frac{\alpha_2}{1 + 2\alpha_2}$.
• when $\alpha_2 = \beta_2 = 0$, $s_2^* = 0$ R accepts any offer

• when $\alpha_2$ is high, $s_2^* \sim 0.5$ fairness is really important (at least not being behind is), R accepts only if 50/50

Proposer’s decision

• if $s < s_2^*$, R rejects then $U_1 = 0$

• if $s \geq s_2^*$, the payoffs are $x_1 = 1 - s$ and $x_2 = s$
\[ U_1 = 1 - s - \alpha_1(s - 1 + s)^+ - \beta_1(1 - s - s)^+ \]

\[ = \begin{cases} 
1 - s - \alpha_1(2s - 1) & \text{if } \frac{1}{2} \leq s \\
1 - s - \beta_1(1 - 2s) & \text{if } \frac{1}{2} \geq s
\end{cases} \]

\[ = \begin{cases} 
(1 + \alpha_2)s - \alpha_2 & \text{if } \frac{1}{2} \leq s \\
(1 - 2\beta_2)s + \beta_2 & \text{if } \frac{1}{2} \geq s
\end{cases} \]
\[ \begin{align*}
\beta_1 > .5 & \quad s = .5 \quad \text{R accepts} \\
\beta_1 < .5 & \quad s = s_2^* = \frac{\alpha_2}{1+2\alpha_2} \quad \text{R accepts}
\end{align*} \]
Remark: Empirically $s^* \sim 1/3$ this implies $\alpha_2 \sim 1$ which means same weight on own wealth than on relative wealth with wealthier people.

Proposition 1: In the market game with n-1 proposers, the equilibrium is $s^* = 1$.

Proposition 2: In the market game with n-1 receivers, it exists an equilibrium with $s^* = 0$. 
1.5 Cooperation and Retaliation

(Public Good Games or Cooperation Games)

1. Game 1: "Pure public good game"

- n players
- player $i$ contributes $g_i$ to the public good
- monetary payoffs

$$x_i = 1 - g_i + a \sum_j g_j$$

with $a \in (\frac{1}{n}, 1)$
• if people are not altruistic $\alpha_i = \beta_i = 0$
  
  – individual rationality

  $$\frac{\partial x_i}{\partial g_i} = -1 + a < 0 \implies g_i^* = 0 \implies x_i^* = 1$$

  – social optimal

  $$S = \sum_j x_j$$

  $$\frac{\partial S}{\partial g_i} = \sum_j \frac{\partial x_j}{\partial g_i} = na - 1 > 0 \implies g_i^c = 1 \implies x_i^c = na$$
2. Game 2: Public good game with punishment.

- everything is public knowledge
- player $i$ can punish player $j$ by an amount $p_{ij}$ with cost $c.p_{ij}$ with $c \in (0, 1)$

3. Empirically

- game 1: people contribute 0
- game 2: people contribute 1 and get punished if they do not do so

4. Predicted by the Fehr-Schmidt model