**Problem 1.1**  (*Phase Transitions in the Erdős-Rényi Model*)

Consider an Erdős-Rényi random graph $G(n, p)$.

(a) Let $A_l$ denote the event that node 1 has at least $l \in \mathbb{Z}^+$ neighbors. Do we observe a phase transition for this event? If so, find the threshold function and justify your reasoning.

(b) Let $B$ denote the event that a cycle with $k$ edges (for a fixed $k$) emerges in the graph. Do we observe a phase transition for this event? If so, find the threshold function and justify your reasoning.

**Problem 1.2**  (*Problem 1.2 from Jackson*)


**Problem 1.3**  (*Clustering in the Configuration Model*)

(a) Consider a graph $g$ with $n$ nodes generated according to the configuration model with a particular degree distribution $P(d)$. Show that the overall clustering coefficient is given by

$$Cl(g) = \frac{\langle d \rangle}{n} \left[ \frac{\langle d^2 \rangle - \langle d \rangle}{\langle d \rangle^2} \right]^2,$$

where $\langle d \rangle$ is the expected degree under distribution $P(d)$, i.e., $\langle d \rangle = \sum dP(d)$ and similarly $\langle d^2 \rangle = \sum d^2P(d)$.

(b) *(Optional for Bonus)*: Consider a power-law degree distribution $P(d)$ given by

$$P(d) = cd^{-\alpha} \quad \text{for } \alpha < 3.$$

Show that the overall clustering coefficient satisfies

$$Cl(g) \sim n^{-\beta}, \quad \beta = \frac{3\alpha - 7}{\alpha - 1}.$$

Discuss the monotonicity properties of the overall clustering coefficient as a function of $n$ for different values of $\alpha$.

**Problem 1.4**  (*Clustering in the Small World Model*)

(a) Consider the small-world model of Watts and Strogatz with rewiring probability $p$. Show that when $p = 0$, the overall clustering coefficient of this graph is given by

$$Cl(g) = \frac{3k - 3}{4k - 2}.$$

(b) *(Optional for Bonus)*: Show that when $p > 0$, the overall clustering coefficient is given by

$$Cl(g) = \frac{3k - 3}{4k - 2} (1 - p)^3.$$