Consider a one-to-one matching market, where $M$ is the set of men and $W$ is the set of women, and strict preferences for the men and women are given. Let $\mu$ and $\mu'$ be any two stable matchings. Let $\mu \lor^M \mu'$ be the function from $M \cup W$ to $M \cup W$ that assigns to each man $m$ the more preferred of $\mu(m)$ and $\mu'(m)$, and assigns to each woman $w$ the less preferred of $\mu(w)$ and $\mu'(w)$. Similarly, let $\mu \land^M \mu'$ be the function that assigns to each man $m$ the less preferred of $\mu(m)$ and $\mu'(m)$, and assigns to each woman $w$ the more preferred of $\mu(w)$ and $\mu'(w)$. We saw in the matching theory slides (Theorem 5) that $\mu \lor^M \mu'$ and $\mu \land^M \mu'$ are again matchings, and moreover are stable.

Now suppose we have any collection $S = \{\mu_1, \ldots, \mu_k\}$ of stable matchings. Define $\sup^M(S)$ to be the function from $M \cup W$ to $M \cup W$ that assigns to each man $m$ the most preferred of $\mu_1(m), \ldots, \mu_k(m)$, and assigns to each woman $w$ the least preferred of $\mu_1(w), \ldots, \mu_k(w)$. We can see that

$$\sup^M(S) = (\cdots ((\mu_1 \lor^M \mu_2) \lor^M \mu_3) \lor^M \cdots) \lor^M \mu_k$$

and therefore, by the preceding result, $\sup^M(S)$ is again a stable matching. Similarly, we can define $\inf^M(S)$ to be the function from $M \cup W$ to $M \cup W$ that assigns to each man $m$ the least preferred of $\mu_1(m), \ldots, \mu_k(m)$, and assigns to each woman $w$ the most preferred of $\mu_1(w), \ldots, \mu_k(w)$. Then $\inf^M(S)$ is again a stable matching.

This leads to the following result. The theorem is due to Teo and Sethuraman (1998), but our exposition follows the approach of Klaus and Klijn (2006).
Theorem 1. Let $\mu_1, \ldots, \mu_l$ be stable matchings, not necessarily distinct, and let $k$ be any integer with $1 \leq k \leq l$. Consider the function $\nu : M \cup W \to M \cup W$ given as follows. For each man $m$, order the matches $\mu_1(m), \ldots, \mu_l(m)$ from most to least preferred (there may be some repetitions in this list); let $\nu(m)$ be the $k$th entry in this list. For each woman $w$, order the matches $\mu_1(w), \ldots, \mu_l(w)$ from most to least preferred, and let $\nu(w)$ be the $(l - k + 1)$th entry in this list. Then $\nu$ is also a stable matching.

For a proof, note that $\mu_1 := \sup M(\{\mu_1, \ldots, \mu_l\})$ has the desired property for $k = 1$, $\mu_2 := \sup M(\{\mu_1, \ldots, \mu_l\} \setminus \{\mu_1\})$ has the desired property for $k = 2$, and so on.

If $\{\mu_1, \ldots, \mu_l\}$ is the set of stable matchings and $l$ is odd, then applying the theorem for $k = (l + 1)/2$ we obtain the median matching, in which every agent is assigned to the median partner over all stable matchings. This formally expresses the idea that we can choose a stable matching that balances the interests of men and women. If $l$ is even, then there are two “almost-median” stable matchings, given by $k = l/2$ and $k = l/2 + 1$. 